Theorie der Kondensierten Materie II SS 2017

PD Dr. B. Narozhny
M.Sc. M. Bard

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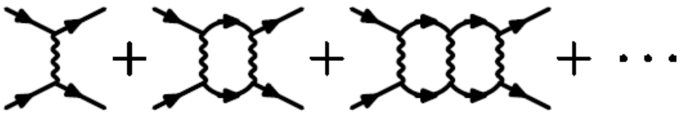
1. Cooper Instability: (25 + 25 + 25 + 25 = 100 Punkte)

Consider the scattering amplitude in the Cooper channel using the Matsubara technique. The corresponding diagrams are shown in the figure.



(a) For the BCS model interaction, the first diagram for the scattering amplitude is given by (the Greek indices refer to the spin components of the incoming and outgoing electrons)

$$\Gamma^{(0)}_{\alpha,\alpha';\beta,\beta'} = \frac{\lambda}{2}\left(\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'}\right).$$

Calculate the second diagram and show that it contains a logarithmic divergence at small frequencies. Show, that higher order diagrams represent a geometric series. Sum up the series and show that the resulting scattering amplitude contains a pole. What is the physical meaning of this pole?

(b) Consider now the generalized susceptibility in the Cooper channel, that can be found by "closing" the end lines of the diagrams for the scattering amplitude. Show that the above instability occurs at low temperatures. Calculate the susceptibility and use your result to find the superconducting transition temperature.

(c) Consider now a more general model, where each wavy line corresponds to

$$\Gamma^{(0)}(i\epsilon, i\epsilon'),$$

where the incoming electrons have the energy $\pm\epsilon$ and the outgoing electrons have the energy $\pm\epsilon'$. Here we neglect the momentum dependence of the scattering amplitude. Derive the integral equation for the scattering amplitude $\Gamma^{(C)}$ in the Cooper chanel within this model.

(d) Consider a specific toy model where

$$\Gamma^{(0)}(i\epsilon, i\epsilon') = \lambda v(i\epsilon) v(i\epsilon').$$

Here we have omitted the spin structure of the scattering amplitude. Find the temperature at which $\Gamma^{(C)}$ diverges. In the case

$$v(i\epsilon) = \frac{\omega_D}{\sqrt{\epsilon^2 + \omega_D^2}},$$

find the relation between T_c and ω_D. Discuss the physical meaning of this model and of the obtained results.