Karlsruher Institut für Technologie
Institut für Theorie der Kondensierten Materie

Theorie der Kondensierten Materie II SS 2017

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1. Coulomb interaction and superconductivity:

Consider the issue of the relative importance of the Coulomb interaction in superconductors. Here we will discuss a simplified model, where the scattering amplitude in the Cooper channel has the form

$$\Gamma^{(0)}(i\epsilon, i\epsilon') = \lambda v(i\epsilon)v(i\epsilon') - \mu.$$

The quantity μ characterizes repulsion between electrons.

In the first approximation, μ is simply the average value of the Coulomb potential in the s-channel. For screened Couomb interaction $V(q) = 4\pi e^2/(q^2 + \varkappa^2)$, with $\varkappa^2 = 8\pi\nu_0 e^2$, the average value is $\mu = \langle V(q)\rangle = (\pi e^2/p_F^2)\ln(1 + 4p_F^2/\varkappa^2)$.

(a) Find the critical temperature in the above model.
Hint: Use the integral equation for the scattering amplitude in the Cooper channel derived in Exercise 9.

(b) Show, that the effect of the Coulomb repulsion is limited to the renormalization of the effective interaction constant

$$\lambda^* = \lambda - \frac{\mu}{1 + \nu_0\mu\ln(E_F/\omega_D)}.$$

2. Josephson effect:

Consider a contact between two superconductors separated by a thin insulating layer. According to quantum mechanics, electrons can tunnel between the two superconductors. The simplest model describing this effect is the so-called tuneling Hamiltonian

$$H_T = \sum_{\boldsymbol{p},\boldsymbol{k},\sigma} \left[T_{\boldsymbol{pk}}\hat{a}^\dagger_{\boldsymbol{p},\sigma}b_{\boldsymbol{k},\sigma} + h.c.\right].$$

The total current flowing from left to right is equivalent to the rate of decrease of the number of electrons in the left metal

$$J = -e\dot{N}_L.$$

(a) Express the tunneling current J in terms of the electronic creation and annihilation operators.

(b) Show that the quantum-mechanical average of the current in the superconducting ground state is equal to zero.

(c) Find a first-order correction to the superconducting ground state due to the tunneling Hamiltonian.

(d) Use the above corection to find the leading contribution to the tunneling current. Express your result in terms of the matrix elements of the tunneling Hamiltonian and the electronic creation and annihilation operators.

(e) Use the Bogoliubov transformation to evaluate the obtained experssion. Recall that the coherence factors u and v are not necessarily real. Express the resulting current in terms of the phases of the order parameters of the two superconductors.