Theorie der Kondensierten Materie II SS 2017

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1. Ruderman-Kittel effect: (40 Punkte)

Consider a localized spin immersed into a free Fermi-gas. The spin interacts with the local electronic spin density by means of the Hamiltonian

$$\hat{H}_{int} = J\hat{S}^i\hat{\sigma}^i(\vec{r}=0),$$

where the local spin density is given by

$$\hat{\sigma}^i(\vec{r}) = \psi^\dagger_\alpha(\vec{r})\sigma^i_{\alpha\beta}\psi_\beta(\vec{r}),$$

and $\sigma^{x,y,z}_{\alpha\beta}$ are the Pauli matrices.

Find the average spin polarization in the electronic system

$$\sigma^i(\vec{r}) = \langle\hat{\sigma}^i(\vec{r})\rangle,$$

at large distances away from the impurity spin (i.e., for $k_F r \gg 1$).

Show that the polarization oscillates as a function of r and find the oscillation period. J can be assumed to be small.

Hint: Use the coordinate representation for the electronic Green's functions.

2. Dynamical spin susceptibility: (60 Punkte)

Find the paramagnetic contribution to the electronic spin susceptibility $\chi(\omega, k)$ at $T = 0$. The spin susceptibility describes the response of the electronic system to an external magnetic field. Consider the limit $\omega \ll E_F$, $k \ll k_F$.

Verify that in the limit $\omega/k \to 0$, $k \to 0$ you recover the Pauli susceptibility. Discuss the importance of the proper limiting procedure and the order of limits.

Solve the problem in two ways - first, by a direct evaluation of the corresponding diagram, and second, by finding the imaginary part of $\chi(\omega, k)$ first, and then restoring the real part using the Kramers-Kronig relations.

Hint: To find the diagram for the spin susceptibility, use the general Kubo formula discussed in the lecture. Recall, that the spin susceptibility describes the response of the electronic spins to an external spins which itself couples to the spins.