Theorie der Kondensierten Materie II SS 2017

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1. Polarons (20 + 20 + 20 = 60 Punkte)

We consider electrons in the conduction band of a semiconductor. The dispersion relation is $E(\vec{p}) = (\vec{p})^2/2m$, where m is the effective (band) mass and the energy is measured from the bottom of the conduction band. The electronic gas in the conduction band is non-degenerate, i.e., the chemical potential is in the gap between the valence and the conduction bands, i.e., $\mu < 0$.

Consider a situation in which electrons interact only via emission and absorption of virtual phonons (no direct Coulomb interaction). Effectively this means that the "wavy" line in our diagrammatic expansion is now replaced by a phononic line. The latter is proportional to the phonon Green's function:

$$U(\omega, \vec{q}) = g^2 \frac{\omega_0^2(\vec{q})}{\omega^2 - \omega_0^2(\vec{q}) + i0} \ .$$

Only acoustic phonons with the dispersion relation $\omega_0(\vec{q}) = c|\vec{q}|$ and $|\vec{q}| < q_D$ are taken into account. Here c is the sound velocity, q_D is the Debye momentum, and g is the coupling constant (deformation potential).

Thus far we considered only direct instantaneous interaction: $U(\vec{r}_1 - \vec{r}_2, t_1 - t_2) = V(\vec{r}_1 - \vec{r}_2)\delta(t_1 - t_2) \to U(\omega, \vec{q}) = V(\vec{q})$. *In contrast the line due to phonons describes interaction with retardation and is* ω *dependent.*

(a) Calculate the lowest order contribution to the self-energy of the electrons, $\Sigma(\epsilon, \vec{p})$. The resulting Green's function describes now polarons (electrons dressed by phonons).

(b) From $\mathrm{Re}\Sigma(\epsilon, \vec{p})$ extract the dispersion relation of the polaron. Find the binding energy and the effective mass of the polaron.
Hint: show that near the mass shell ($\epsilon \approx E(\vec{p}) - \mu$*) and for* $|\vec{p}| \ll mc$ *the self energy reads*

$$\Sigma(\epsilon, \vec{p}) = \epsilon_0 - \alpha_1 \left[\epsilon + \mu - E(\vec{p})\right] - \alpha_2 E(\vec{p}).$$

(c) Consider $\mathrm{Im}\Sigma(\epsilon, \vec{p})$ and find the life-time of a polaron with momentum $\vec{p}$.

2. Jordan-Wigner Transformation: (20 + 20 = 40 Punkte)

(a) Consider the set of Pauli matrices σ_n^α satisfying the usual SU(2) commutation relations for each n, but commuting for different n.

Show that the following transformation:

$$\sigma_n^z = 2a_n^\dagger a_n - 1, \quad \sigma_n^- = a_n \prod_{m<n} \sigma_m^z, \quad \sigma_n^+ = a_n^\dagger \prod_{m<n} \sigma_m^z,$$

maps the above set of Pauli matrices into a set of fermionic operators.

Show that the operators a_n defined by the transformation obey the fermionic commutation relations for each n and anticommute for different n.

(b) Consider the one-dimensional spin chain, described by the generic Hamiltonian

$$\hat{H} = \sum_{n=-\infty}^{\infty} \left(J_x \sigma_n^x \sigma_{n+1}^x + J_y \sigma_n^y \sigma_{n+1}^y + J_z \sigma_n^z \sigma_{n+1}^z - B\sigma_n^z \right),$$

where σ_n^α are the Pauli matrices.

Use the above Jordan-Wigner transformation to express $\hat{H}$ in terms of fermions.