Karlsruher Institut für Technologie

Institut für Theorie der Kondensierten Materie

Theorie der Kondensierten Materie II SS 2017

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Blatt 7

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1. Ruderman-Kittel effect at $T > 0$: (30 Punkte)

Solve the problem 1 of Blatt 5 at $T > 0$.

2. Analytic continuation and Matsubara susceptibility ($40 + 30 = 70$ Punkte)

In the lecture we have introduced the Kubo formula, which gives the linear response function (susceptibility) as

$$\chi_{BA}(t - t') = D^R_{BA} = -i\theta(t - t')\langle[B(t)A(t')]\rangle.$$

Here A is the observable to which the external force $f(t)$ is coupled ($H_f = H + f(t)A$), whereas B is the observable that is being measured. The operators are in the Heisenberg representation. In the lecture we have defined the susceptibility at $T = 0$, so that the averaging inthe above equation stands for averaging over the ground state. A straightforward generalization is to define the susceptibility at $T > 0$ by using the thermal averaging instead.

One can introduce also Matsubara susceptibility:

$$\chi^M_{BA}(\tau) = -\langle T_\tau B(\tau)A(0)\rangle,$$

where $\tau \in [-1/T, 1/T]$. The Fourier transform would read

$$\chi^M_{BA}(i\omega_n) = -\frac{1}{2}\int\limits_{-1/T}^{1/T} d\tau \langle T_\tau B(\tau)A(0)\rangle e^{i\omega_n \tau}.$$

Here $\omega_n = 2n\pi T$.

(a) Prove that $\chi_{BA}(\omega)$ is given by an analytic continuation of $\chi^M_{BA}(i\omega_n)$ from positive discreet frequencies onto the real axis, $i\omega_n \to \omega + i0$.

(b) Solve the problem 2, Blatt 5 at $T > 0$.