Karlsruher Institut für Technologie | Institut für Theorie der Kondensierten Materie

**Theorie der Kondensierten Materie II SS 2017**

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**Blatt 9**
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**1. Diffusion:** (25 + 35 + 20 + 20 = 100 Punkte)

For non-interacting electrons in the presence of weak disorder, we consider linear response of the particle density $n(\boldsymbol{r}, t)$ to the weak external potential. The corresponding term in the Hamiltonian is given by

$$H_{int} = -\int d^3 r n(\boldsymbol{r}, t) V(\boldsymbol{r}, t).$$

The correspoding response function in the frequency domain is defined by the relation

$$\langle n(\boldsymbol{r}, \omega) = \int d^3 r' K(\omega, \boldsymbol{r}, \boldsymbol{r}') V(\boldsymbol{r}', \omega).$$

(a) Show that within the Matsubara technique, the density-density response function can be expressed in terms of the exact Green's functions of the disordered system as

$$K(i\omega_m, \boldsymbol{r}, \boldsymbol{r}') = T \sum_n G(i\epsilon_n, \boldsymbol{r}, \boldsymbol{r}') G(i\epsilon_n + i\omega_m, \boldsymbol{r}, \boldsymbol{r}').$$

Draw the corresponding diagram.

(b) Assuming isotropic scattering on impurities, average the above response function over disorder. Show, that the dominant contribution is given by the ladder series of diagrams, desribed in the lecture as the "diffuson" series. Sum up the series in the Matsubara technique, and show that for small momenta and frequencies the density-density response function has a form

$$K(i\omega_m, \boldsymbol{q}) = -\frac{\nu D q^2}{|\omega_m| + D q^2}.$$

Find the expression for the diffusion conefficient $D$ and identify the conditions, under which the above expression is valid.

(c) Compare your results to the classical diffusion equation. Write down the diffusion equation in the presence of an external force and find the classical response function. Show that the result corresponds to the above result of the diagrammatic calculation (after analytic continuation). Consider the diffuson diagrams in real space and explain why do they correspond to diffusion.

(d) Consider the classical diffusion equation and find its Green's function (as the Green's function of the linear differential equation). Use the Green's function of the diffusion equation to find the probability of the particle to return to a given point in space. Explain, how your results can be used to justify the qualitative arguments used in the lecture to desribe the weak localization correction.