Karlsruher Institut für Technologie
Institut für Theorie der Kondensierten Materie

Theorie der Kondensierten Materie I WS 2012/2013

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1. Magnetic analogy in BCS theory (20 + 20 + 20 + 20 + 20 = 100 Punkte)

(a) Show that the isospin operators $\boldsymbol{\tau}_k = \psi_k^\dagger \boldsymbol{\tau} \psi_k$ obey the $SU(2)$ algebra $[\tau_k^\alpha, \tau_k^\beta] = 2i\epsilon^{\alpha\beta\gamma}\tau_k^\gamma$. Here, $\psi_k^\dagger = (c_{k\uparrow}^\dagger, c_{-k\downarrow})$ is the Nambu spinor with electron creation operators $c_{k\sigma}^\dagger$ and $\boldsymbol{\tau} = (\tau^x, \tau^y, \tau^z)$ is a vector of Pauli matrices τ^α.

(b) Rewrite the BCS Hamiltonian

$$H_{BCS}^{MF} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_k \left(\Delta_k^* c_{-k\downarrow} c_{k\uparrow} + \Delta_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger\right) + \sum_k \frac{|\Delta_k|^2}{V_0} \tag{1}$$

in the form of the isospin in a magnetic field $H_{BCS}^{MF} = -\sum_k \boldsymbol{B}_k \cdot \boldsymbol{\tau}_k$. Determine magnitude and direction of the magnetic field $\boldsymbol{B}_k = |\boldsymbol{B}_k| \hat{\boldsymbol{n}}_k$ with unit vector $\hat{\boldsymbol{n}}_k$.

(c) In the ground state at $T = 0$ all isospins are aligned with the field $\boldsymbol{B}_k$. Determine the direction of the spins across the Fermi surface, *i.e.*, for $\epsilon_k < 0$, $\epsilon_k = 0$ and $\epsilon_k > 0$, and draw the spins. Assume a real gap $\Delta_k \in \mathbb{R}$.

(d) Determine the isospin state that corresponds to the electron vacuum $|0\rangle$, *i.e.*, $c_{k\sigma}|0\rangle = 0 \ \forall k, \sigma$. Find the BCS ground state by rotating each spin out of the vacuum configuration to be aligned with the field $\boldsymbol{B}_k$. Again assume a real gap $\Delta_k \in \mathbb{R}$.

(e) Find the two eigenvectors of the Hamiltonian $\boldsymbol{B}_k \cdot \boldsymbol{\tau}_k$ with isospin quantized parallel and anti-parallel to $\hat{\boldsymbol{n}}_k$. Project the Nambu spinor $\psi_k^\dagger$ onto these eigenvectors and rewrite the BCS Hamiltonian in terms of the resulting operators. Determine the ground state energy and the excitation spectrum above the ground state. Excitations can be interpreted as spin flips from a configuration parallel to one anti-parallel to the field $\boldsymbol{B}_k$.