Theorie der Kondensierten Materie I WS 2012/2013

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1. Parity effect. (10 + 30 + 30 + 30 Punkte)

At low temperatures, $T \ll T_c$, only a very small fraction $\propto e^{-\Delta/T}$ of electrons in a superconductor occupy excited states with energies $E > \Delta$, while the other electrons form Cooper pairs. Because of that, the parity of the total number N of electrons in an isolated superconductive grain may be crucially important for its physical properties, even at very large $N \sim 10^9$ [see, e.g., Phys. Rev. Lett. 69, 19972000 (1992)]. Indeed, for an even N, all the electrons in the grain are coupled, and adding one more electron would cost extra energy Δ, compared to the case of an odd N, when there is one unpaired electron. This difference will manifest itself, e.g., when observing transport between two normal electrodes through a superconductive island.

(a) Let us consider first a normal metallic grain with discrete energy levels $\varepsilon_{\mathbf{k}\alpha}$ ($\mathbf{k}$ and α being the momentum and the spin of the respective single-electron state). Derive the grand-canonical potential Ω_{even} and Ω_{odd} at temperature T under the constraints that only even and only odd numbers of electrons in the grain are allowed, respectively. Neglect the Coulomb energy associated with different particle numbers.

(b) Evaluate $\delta\Omega(T) = \Omega_{odd}(T) - \Omega_{even}(T)$ in a large metallic grain of volume V. The density of states close to the Fermi level is ρ_F.
Hint: rewriting the grand-canonical partition functions in the form $[\Pi_i \left(1 + e^{-\beta\varepsilon_i}\right) \pm \Pi_i \left(1 - e^{-\beta\varepsilon_i}\right)]/2$ may be useful.

(c) In a superconductive grain the grand-canonical potential is a sum of the Cooper-pair contribution and the grand-canonical potential of the quasiparticles. The number of electrons in the Cooper-pair condensate is even, so $\delta\Omega(T) = \Omega_{odd}(T) - \Omega_{even}(T)$ is determined by the quasiparticle gas. Evaluate $\delta\Omega(T)$ in a superconductive grain, using the quasiparticle spectrum $E_{\mathbf{k}\alpha} = \sqrt{\Delta^2 + \varepsilon^2_{\mathbf{k}\alpha}}$.

(d) What is the probability of finding an unpaired electron in a superconductive grain at low temperatures? Estimate the characteristic temperature, at which the parity effects vanish.