Karlsruhe Institute of Technology

Winter-Semester 2017/18

Moderne Theoretische Physik IIIa

Statistische Physik

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Do 11:30-13:00, Lehmann Raum 022, Geb 30.22

http://www.tkm.kit.edu/lehre/

Zusammenfassung

$$Z_G = \prod_\lambda Z_\lambda$$

Bosonen $$Z_\lambda = \frac{1}{1 - e^{-\beta(\epsilon_\lambda - \mu)}}$$

Fermionen $$Z_\lambda = 1 + e^{-\beta(\epsilon_\lambda - \mu)}$$

$$\langle n_\lambda \rangle = \left| \begin{array}{l} n_B(\epsilon_\lambda) = \dfrac{1}{e^{\beta(\epsilon_\lambda - \mu)} - 1} \\[2em] n_F(\epsilon_\lambda) = \dfrac{1}{e^{\beta(\epsilon_\lambda - \mu)} + 1} \end{array} \right.$$

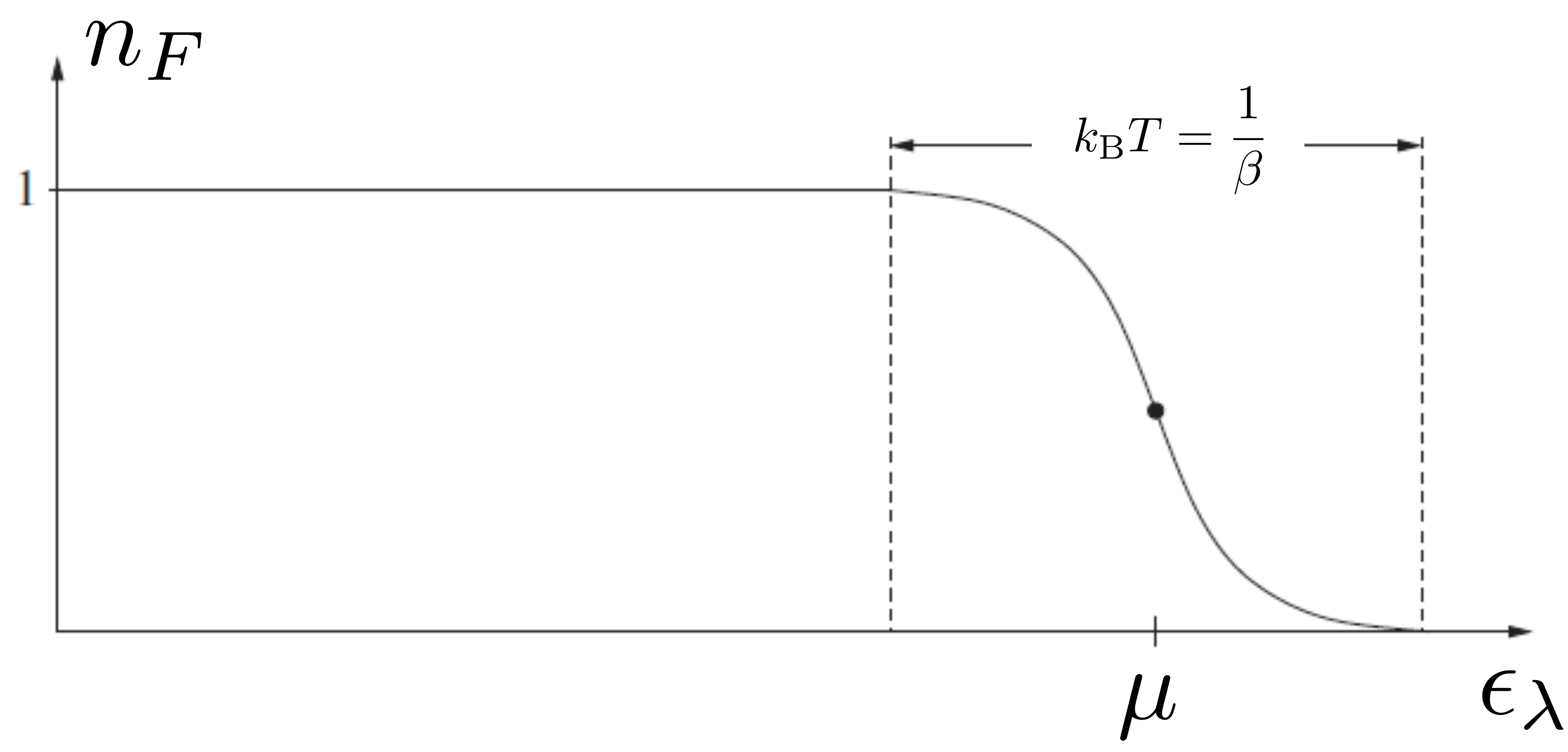
Bose-Funktion

Fermi-Funktion

Ideales Fermi-Gas

$$Z_G(T,V,\mu) = \prod_\lambda Z_\lambda = \prod_\lambda \left[1 + e^{-\beta(\epsilon_\lambda - \mu)}\right]$$

Fermi-Funktion $$\langle n_\lambda \rangle = n_F(\epsilon_\lambda) = \frac{1}{e^{\beta(\epsilon_\lambda - \mu)} + 1}$$



Ideales Fermi-Gas

$$Z_G(T,V,\mu) = \prod_\lambda \left[1 + e^{-\beta(\epsilon_\lambda - \mu)}\right]$$

$$\Omega(T,V,\mu) = -k_\mathrm{B} T \ln Z_G(T,V,\mu) = -k_\mathrm{B} T \sum_\lambda \ln\left[1 + e^{-\beta(\epsilon_\lambda - \mu)}\right]$$

$$\langle N \rangle = -\frac{\partial \Omega}{\partial \mu}\Big|_{T,V} = \sum_\lambda \frac{e^{-\beta(\epsilon_\lambda - \mu)}}{1 + e^{-\beta(\epsilon_\lambda - \mu)}} = \sum_\lambda \langle n_\lambda \rangle$$

μ gegeben $\rightarrow N$ (großkanonisch)
N gegeben $\rightarrow \mu$ (kanonisch)

Berechnet wird (fast) immer großkanonisch, auch wenn N gegeben ist

Ideales Fermi-Gas

Übung: $\langle n_\lambda^2 \rangle = \langle n_\lambda \rangle$ und $\langle n_{\lambda_1} n_{\lambda_2} \rangle = \langle n_{\lambda_1} \rangle \langle n_{\lambda_2} \rangle$

$N = \sum_\lambda n_\lambda$ ⇨ $(\Delta N)^2 = \langle N^2 \rangle - \langle N \rangle^2 \leq \langle N \rangle$

$$\frac{\Delta N}{\langle N \rangle} \leq \frac{1}{\sqrt{\langle N \rangle}} \to 0 \text{ für } N \to \infty$$

Ideales Fermi-Gas

$$\Omega(T, V, \mu) = -k_\mathrm{B} T \sum_\lambda \ln\left[1 + e^{-\beta(\epsilon_\lambda - \mu)}\right]$$

$$\langle n_\lambda \rangle = n_F(\epsilon_\lambda) = \frac{1}{e^{\beta(\epsilon_\lambda - \mu)} + 1}$$

$$S = -\frac{\partial \Omega}{\partial T}\bigg|_{V,\mu} = -k_\mathrm{B} \sum_\lambda \left[\langle n_\lambda \rangle \ln \langle n_\lambda \rangle + (1 - \langle n_\lambda \rangle) \ln(1 - \langle n_\lambda \rangle)\right]$$

$$S = -k_\mathrm{B} \sum_\lambda \left[n_F(\epsilon_\lambda) \ln n_F(\epsilon_\lambda) + (1 - n_F(\epsilon_\lambda)) \ln(1 - n_F(\epsilon_\lambda))\right]$$

$S(T \to 0) \to 0$ 3. Hauptsatz

Ideales Fermi-Gas

$$\Omega(T, V, \mu) = -k_{\mathrm{B}} T \sum_{\lambda} \ln\left[1 + e^{-\beta(\epsilon_\lambda - \mu)}\right]$$

$$N = \sum_{\lambda} n_F(\epsilon_\lambda) \qquad n_F(\epsilon_\lambda) = \frac{1}{e^{\beta(\epsilon_\lambda - \mu)} + 1}$$

$$S = -k_{\mathrm{B}} \sum_{\lambda} \left[n_F(\epsilon_\lambda) \ln n_F(\epsilon_\lambda) + (1 - n_F(\epsilon_\lambda)) \ln(1 - n_F(\epsilon_\lambda))\right]$$

$$U = \Omega + TS + \mu N = \sum_{\lambda} \epsilon_\lambda n_F(\epsilon_\lambda)$$

$$P = -\frac{\partial \Omega}{\partial V}\bigg|_{T,\mu} = -\frac{\Omega}{V}$$

Ideales Fermi-Gas, T=0

$$n_F(\epsilon_\lambda) = \frac{1}{e^{\beta(\epsilon_\lambda - \mu)} + 1} = \theta(\mu - \epsilon_\lambda) = \begin{cases} 1 & \text{für} \quad \epsilon_\lambda < \mu \\ 0 & \text{für} \quad \epsilon_\lambda > \mu \end{cases}$$

$$\Omega(T, V, \mu) = -k_\mathrm{B} T \sum_\lambda \ln\left[1 + e^{-\beta(\epsilon_\lambda - \mu)}\right] = \sum_\lambda (\epsilon_\lambda - \mu)\theta(\mu - \epsilon_\lambda)$$

$$N = \sum_\lambda n_F(\epsilon_\lambda) = \sum_\lambda \theta(\mu - \epsilon_\lambda) \qquad \Delta N = 0$$

$$\mu \text{ gegeben} \rightarrow N \text{ (großkanonisch)}$$
$$N \text{ gegeben} \rightarrow \mu \text{ (kanonisch)}$$

Berechnet wird (fast) immer großkanonisch, auch wenn N gegeben ist

Ideales Fermi-Gas, T=0

$$n_F(\epsilon_\lambda) = \frac{1}{e^{\beta(\epsilon_\lambda - \mu)} + 1} = \theta(\mu - \epsilon_\lambda) = \begin{cases} 1 & \text{für} \quad \epsilon_\lambda < \mu \\ 0 & \text{für} \quad \epsilon_\lambda > \mu \end{cases}$$

$\lambda = \mathbf{p}, \sigma$ z.B. Elektronen (Spin s=1/2) $\sigma = -s, -s+1, \ldots, s-1, s$

$$\epsilon_\lambda = \epsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m}$$

$$N = \sum_\lambda \theta(\mu - \epsilon_\lambda) = (2s+1)V \int d\epsilon\, \nu(\epsilon)\, \theta(\mu - \epsilon)$$

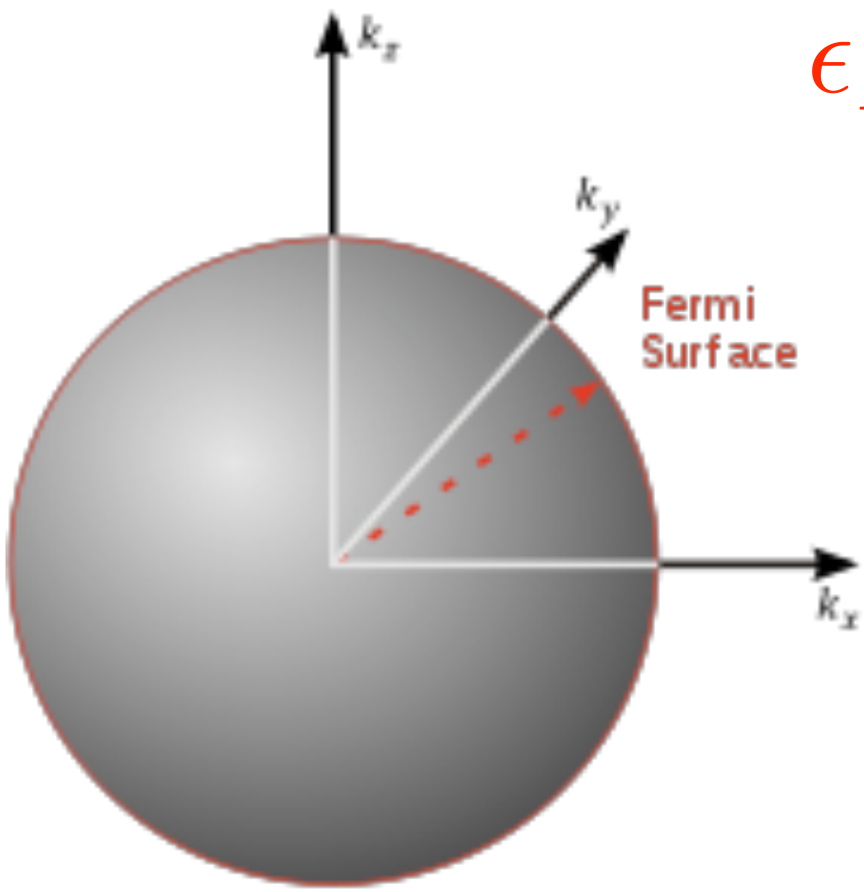
$$\nu(\epsilon) = \mathcal{N}(\epsilon) = D(\epsilon) = \frac{1}{(2s+1)V} \sum_\lambda \delta(\epsilon - \epsilon_\lambda) = \frac{1}{V} \sum_{\mathbf{p}} \delta(\epsilon - \epsilon_{\mathbf{p}})$$

Zustandsdichte pro Volumen und Spin

Ideales Fermi-Gas, T=0

$$n = \frac{N}{V} = (2s+1) \int d\epsilon\, \nu(\epsilon)\, \theta(\mu - \epsilon)$$

Die Teilchen-Dichte n ist gegeben $\rightarrow \mu(T=0) \equiv \epsilon_F$



ϵ_F - Fermi-Energie

$$\epsilon_F = \frac{p_F^2}{2m}$$

$$p_F = \sqrt{2m\epsilon_F}$$

Fermi-Impuls

Fermi-See

Ideales Fermi-Gas, T=0

Kasten L_x, L_y, L_z **Zustände** $(p_x, p_y, p_z) = \left(\frac{2\pi\hbar}{L_x}n_x, \frac{2\pi\hbar}{L_y}n_y, \frac{2\pi\hbar}{L_z}n_z\right)$

$$\frac{d^3p}{(2\pi\hbar)^3} = \nu(\epsilon)d\epsilon \quad \Rightarrow \quad \nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$$

$$\begin{aligned} n = \frac{N}{V} &= (2s+1)\int\frac{d^3p}{(2\pi\hbar)^3}\theta(\epsilon_F - \epsilon_{\mathbf{p}}) \\ &= \frac{(2s+1)}{(2\pi\hbar)^3}\cdot\frac{4}{3}\pi p_F^3 \end{aligned}$$

$$\Rightarrow \quad \mu(T=0) = \epsilon_F = \frac{\hbar^2}{2m}\left(\frac{6\pi^2 n}{2s+1}\right)^{\frac{2}{3}}$$

Typisch

$$\epsilon_F \approx 10\text{eV}$$

$$T_F \equiv \frac{\epsilon_F}{k_\text{B}} \approx 10^5\text{K}$$