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THEORETICAL OPTICS: EXERCISE SHEET 8

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1. Optical carpets - Talbot effect

Consider the diffraction of electromagnetic plane waves of wave-length $\lambda = 2\pi/k$, on an infinite periodic grating placed at $z = 0$ that leads to the following modulated profile for the electric field

$$\mathcal{E}(y, z = 0) = \mathcal{E}_0 \left[1 + m \cos\left(\frac{2\pi y}{L}\right)\right] , \tag{1}$$

where $\mathcal{E}_0$,m and L are constants. The grating extends to infinity in the xy-plane, while translational invariance along the $x-$axis renders the problem essentially two-dimensional (only y, z are relevant).

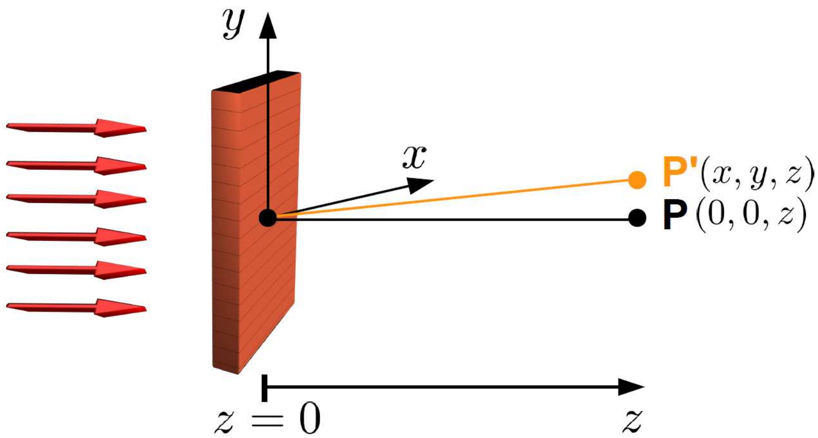

FIG. 1: Diffraction of plane waves on an infinite periodic grating. Within the "near"-field Fresnel approximation, one may retrieve the Talbot effect where for specific distances z, the diffracted pattern is the self-image of the pattern at $z = 0$.

a. Calculate the electric field at the observation point $\boldsymbol{P'}$ depicted in Fig. 1, using the scalar diffraction formula within the Fresnel approximation. Plot the intensity in the yz-plane that defines the optical carpet profile. (5 points)

Hint: You may need the following Fresnel integrals

$$\int_{-\infty}^{+\infty} \cos\left(\frac{\pi u^2}{2}\right) du = 1 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \sin\left(\frac{\pi u^2}{2}\right) du = 1 \,. \tag{2}$$

b. Determine the distances z for which the Talbot effect occurs, i.e. the diffraction pattern is the self-image of the intensity pattern at $z = 0$. (5 points)

2. Diffraction on single and double circular apertures - Airy pattern

Consider a planar electromagnetic wavefront of wave-length $\lambda = 2\pi/k$ diffracted on the single and double circular apertures depicted below

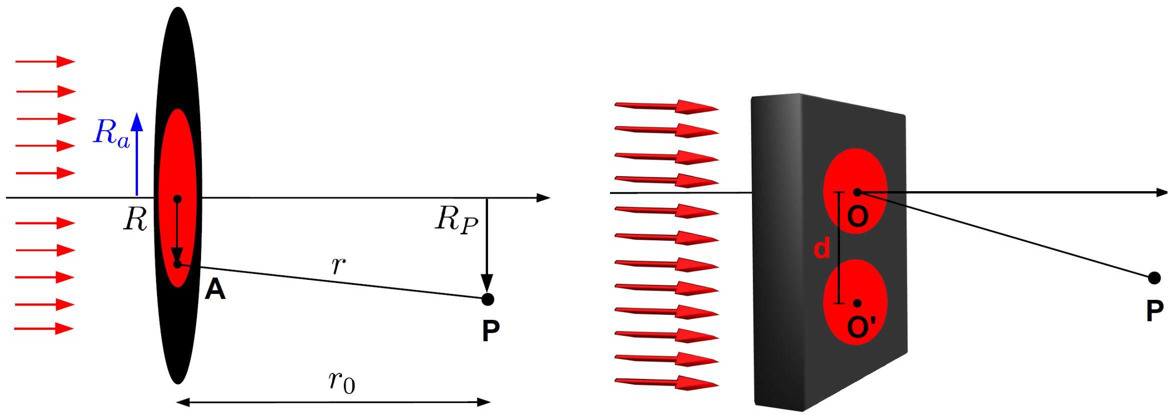

FIG. 2: Diffraction of plane waves on single and double circular apertures.

a. Using the scalar diffraction formula within the Fraunhofer approximation, calculate the electric field at the observation point $\boldsymbol{P}$ of Fig. 2 for a single circular aperture, that leads to the so-called Airy pattern. (4 points)

Hint: For the particular calculation you can make use of the integral representation of the Bessel functions

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{-i(n\tau - x\sin\tau)}d\tau \tag{3}$$

and the relation

$$J_{n-1}(x) = \frac{n}{x}J_n(x) + \frac{dJ_n(x)}{dx}\,. \tag{4}$$

b. The diffraction pattern of **1a.** consists of a central bright disc, called Airy's disc. Determine its area. (2 points)

c. Determine the intensity at an observation point $\boldsymbol{P}$, for a diffraction screen consisting of two shifted identical circular apertures depicted in Fig. 2. Determine the points $\boldsymbol{P}$ for which we obtain interference minima. (4 points)

3. Bragg reflection

Consider the two dimensional cut of a crystal depicted in Fig. 3.

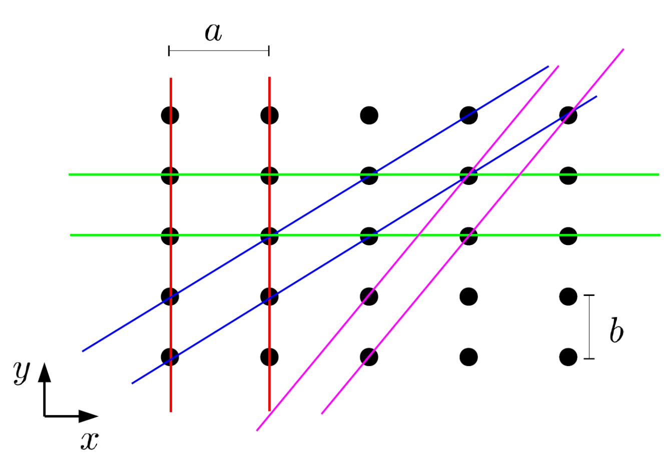

FIG. 3: A two-dimensional cut of a crystal. With several colors we depict the edges of planes that extend infinitely perpendicular to the xy-plane.

a. Calculate the general formula providing the spacing of parallel planes that extend infinitely perpendicular to the xy-plane, for the particular crystal depicted in Fig. 3. (3 points)

b. Determine the condition for observing Bragg reflection maxima. (2 points)

c. For the crystal structure of Fig. 3, consider the Bragg reflection with X-rays of wave-length $\lambda = 1.54\text{Å}$. If $a = 2b = 4\text{Å}$, calculate the Bragg angle for all orders of reflection for the different planes depicted in Fig. 3. (5 points)