Übungen zur Theorie der Kondensierten Materie II SS 13

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1. **Failure of Fermi liquid theory in 1D** (10 Punkte)

The purpose of this exercise is to compute perturbatively the residue of the of fermionic Green function of 1D interacting fermions

$$Z = \frac{1}{1 - \partial_\epsilon \mathrm{Re}\Sigma(p,\epsilon)}\bigg|_{p=p_F,\,\epsilon=\epsilon_F} \tag{1}$$

We will see that $Z = 0$ which signals the failure of Fermi liquid theory in 1D.

(a) Consider spinless interacting fermion with linear dispersion relation at $T = 0$ interacting via point-like density-density interaction with interaction constant g. Draw the three diagrams contributing to the self energy Σ_+ of right-moving particles in the second order in g. Show that the two diagrams involving only the Green functions of right-movers cancel.

(b) Consider the polarization operator (density bubble) for left movers involved in the remainig diagramm for Σ_+. Show that

$$\Pi_-(\omega, q) = \frac{1}{2\pi}\frac{q}{v_F q + \omega + i0\,\mathrm{sign}\omega} \tag{2}$$

(c) Write the analytic expression for the diagramm from (a) and perform the frequency integration. Show that the real part of Σ_+ is given by

$$\mathrm{Re}\Sigma_+(p,\epsilon) = \frac{g^2}{(2\pi)^2}\int_0^\infty dq \left[\frac{q}{\epsilon + v_F(2q-p)} + \frac{q-p}{\epsilon - v_F(2q-p)}\right] \tag{3}$$

(d) Calculate the quasiparticle residue Z.

2. **Dzyaloshinskii-Larkin theorem** (8 Punkte)

The purpose of this exersise is to show that in the Tomonaga-Luttinger model all the loops made out of $n \le 3$ fermionic lines vanish. This means that the RPA approximation is exact.

(a) Let us consider a loop made out of three fermionic Green functions and with three wavy lines as external legs carrying frequencies ω_i and momenta k_i, $i = 1, 2, 3$. Physically, such a diagram represents the cubic interaction of density fluctuations (compare to polarization operator). To be precise, there are two diagrams of this type which differ by the order of wavy lines. Draw these two diagrams and write down the corresponding analytical expressions. Assume Matsubara technique for definiteness.

(b) Use the following simple identity

$$\frac{1}{i\epsilon - qV_F}\frac{1}{i(\epsilon+\omega_1) - V_F(q+k_1)} = \frac{1}{i\omega_1 + V_F k_1}\left[\frac{1}{i\epsilon - V_F q} - \frac{1}{i(\epsilon+\omega_1) - V_F(k_1+q)}\right] \tag{4}$$

to transform the analytic expressions for the diagrams discussed in task (a). What is the graphical representation of this transformation?

(c) Show that the sum of the two diagrams from task (a) vanish.

(d) Generalize the above arguments to the case of arbitrary fermionic loop with more than 2 fermionic lines.

(e) Why the line of reasoning (a)-(c) does not apply to fermionic loop made out of two fermionic lines (polarization operator)?

3. Critical exponents and Anderson transition (8 Punkte)

Disorder driven Anderson metal-insulator transitions (MITs) constitute a (very peculiar) example of (quantum) phase transitions. Generally, the behavior at the transition point of a continuous phase transition is universally described by "critical exponents". The goal of this exercise is to determine the critical exponents from the beta function in the exemplary case of the Anderson MIT.
Consider the beta function $\frac{d\ln g}{d\ln(L/l)} = \beta(g)$ where g denotes the dimensionless conductance, l the UV scale and L the running scale. Assume that beta function crosses zero exactly once at $g = g_c$ and in the vicinity of this point can be Taylor expanded

$$\beta(g) = \beta_c'(g - g_c) + \dots, \qquad \beta_c' > 0 \tag{5}$$

(a) Consider the following situations at the UV scale

$$(i)\ g(l) = g_0 > g_c \text{ or } (ii)\ g(l) = g_0 < g_c.$$

How does $g(L)$ behave in the thermodynamic limit $L \to \infty$ for both cases? What is the meaning of g_c?

(b) Linearize and the renormalization group equation for given $\delta g(l) = \delta g_0 \ll g_c$ and determine the scale ξ at which the linear approximation fails to remain valid. This scale ξ marks the crossover from the critical scale dependence of g observed in the vicinity of g_c to the classical metallic or insulating behavior at $g \to \infty$ or $g \to 0$ (for example, Ohm's law $g = \sigma L^{d-2}$ in the metallic phase). What is the meaning of ξ in insulating phase?

(c) The critical exponent ν governs the behaviors of the correlation length in the vicinity of the critical point, $\xi \sim l_0|\delta g_0|^{-\nu}$. Read off the exponent ν from the results obtained in (b).