Karlsruher Institut für Technologie
Institut für Theorie der Kondensierten Materie

Übungen zur Theorie des Magnetismus   Sommersemester 16

PD Dr. B. Narozhny
T. Ludwig

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1. Magnon dispersion derived with use of Bloch states (5 Punkte)

In the lectures, the magnon dispersion was derived with the Holstein-Primakoff transformation. In this exercise, we consider another method to obtain the Magnon dispersion relation.

(a) Derive the magnon dispersion by constructing a Bloch wave of single spin-flips and showing that it is an eigenstate to the Hamiltonian. Use the anisotropic Heisenberg Hamiltonian with external magnetic field.

(b) Discuss the isotropic case with and without external field and also the anisotropic case.

2. Magnon dispersion from semi-classical dynamics (5 Punkte)

In this exercise, we will consider another way to obtain the magnon dispersion, this time by considering the semi-classical dynamics of the spins.

(a) Write down the Heisenberg equations of motion for the isotropic Heisenberg Hamiltonian with external field. Rewrite the equations, such that the torque on the spin at each side is given by an effective magnetic field.

(b) Write down the equations of motion for each spin-component. Now, consider the case of a ferromagnet and assume that the magnetic field is along the z-direction. Simplify the equations of motion by considering low-energy excitations only, i.e. assume that the value of S_z is always close to its maximum.

(c) It is useful to combine the equations of motion for S_{jx}, S_{jy} into equations for $S_{j\pm} = S_{jx} \pm iS_{jy}$. Assume that the spins S_j sit on a Bravais lattice and perform a Fourier transform.

(d) Now, use the ansatz $S_{k-} \propto \delta S_{k-} \mathrm{e}^{-i\omega_k t + i\alpha}$ and determine the behavior of ω_k for small values of k.