

Übungen zur Theorie des Magnetismus Sommersemester 16PD DR. B. NAROZHNY
T. LUDWIG**Blatt 5**
Besprechung 24.6.2016**1. 4-Site Heisenberg model** (5 Punkte)

In this exercise, we consider the antiferromagnetic Heisenberg model with four sites and $S = \frac{1}{2}$, the Hamiltonian is given by

$$H_4 = J (\mathbf{S}_1\mathbf{S}_2 + \mathbf{S}_2\mathbf{S}_3 + \mathbf{S}_3\mathbf{S}_4 + \mathbf{S}_4\mathbf{S}_1) \quad (1)$$

- (a) What is the nature of the ground state? Does it show Néel-type order, or at least antiferromagnetic correlations?
- (b) What are the level degeneracies? Do they change if we include the next-nearest neighbour couplings in

$$H'_4 = H_4 + J' (\mathbf{S}_1\mathbf{S}_3 + \mathbf{S}_2\mathbf{S}_4) \quad (2)$$

2. Scattering amplitudes in the XXZ-model (5 Punkte)

In the lectures, the Bethe ansatz was considered. This exercise is about filling in the intermediate steps in the derivation of the scattering amplitudes for the $M = 2$ case.

- (a) Using the Bethe ansatz, solve the eigenvalue problem to obtain the scattering amplitudes in terms of Δ, k_1, k_2 .