Karlsruher Institut für Technologie

Institut für Theorie der Kondensierten Materie

Übungen zur Theorie der Kondensierten Materie II SS 16

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Blatt 3

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1. Density-density response in one dimension (8 Points)

In this problem we derive the Lindhard function in one spatial dimension. The density-density response of a free-electron gas is given in momentum space by the relation

$$\chi_q^r(\omega) = \frac{1}{L}\sum_k f_k \left(\frac{1}{\omega + i0^+ - \epsilon_k + \epsilon_{k-q}} - \frac{1}{\omega + i0^+ + \epsilon_k - \epsilon_{k+q}} \right), \tag{1}$$

where f_k is the Fermi-Dirac distribution and L is the length of the system. The energies ϵ_k are the energies of free particles

$$\epsilon_k = \frac{k^2}{2m} - \mu. \tag{2}$$

(a) Replace the sum in (1) by an integral and determine Re $\left[\chi_q^r(\omega)\right]$ and Im $\left[\chi_q^r(\omega)\right]$ in one dimension for $T = 0$. (6 Pts.)

(b) As discussed in the lecture, if the dielectric function $\epsilon(q,\omega)$ vanishes for certain pairs of values of (q,ω), the system reacts strongly to an arbitrarily small external field component with the same (q,ω). These excitations are called plasmons. Using the connection between the dielectric function and the density-density response, find the dispersion-relation $\omega(q)$ for these excitations. (2 Pts.)

2. Thomas-Fermi Theory (10 Points)

A simple form of the static dielectric function $\epsilon(\boldsymbol{q})$ is obtained in the Thomas-Fermi theory of screening. Here we start again with an extra charge density ρ^{ext} and an induced charge density ρ^{ind} that produce the full potential φ. The Poisson equation relates these quantities:

$$\nabla^2\varphi(\boldsymbol{r}) = -4\pi\left[\rho^{\text{ext}}(\boldsymbol{r}) + \rho^{\text{ind}}(\boldsymbol{r})\right], \tag{3}$$

where the problem is assumed to be static, thus there is no time-dependence of the various quantities. The extra charge density can be written via the difference between the equilibrium particle density n_0 and the local particle density $n(\boldsymbol{r})$:

$$\rho^{\text{ind}}(\boldsymbol{r}) = -e\left[n(\boldsymbol{r}) - n_0\right] \tag{4}$$

(a) We now assume that the potential φ is only slowly varying in space. Then we can define regions in which the potential is approximately constant. This allows us to treat these regions as a free electron gas with a Fermi wave-vector k_F that varies from region to region. The value of k_F is fixed by demanding that the chemical potential μ is a constant throughout the material. This gives the condition

$$\frac{k_F^2(\boldsymbol{r})}{2m} + e\varphi(\boldsymbol{r}) = \mu. \tag{5}$$

From these relations derive a differential equation that governs the potential $\varphi(\boldsymbol{r})$ with $\rho^{\text{ext}}(\boldsymbol{r})$ as source-term. You will have to use the well-known relation between particle density n and wave-vector k_F from the theory of the free electron gas. (4 Pts.)

(b) This differential equation contains a non-linear term in $\varphi(\boldsymbol{r})$. In order to make progress, linearize this term by assuming that $\varphi/\mu \ll 1$. The resulting equation is of the form

$$(\nabla^2 - q_F^2)\varphi(\boldsymbol{r}) = -4\pi\rho^{\text{ext}}(\boldsymbol{r}). \tag{6}$$

Give the expression for q_F. (1 Pt.)

(c) Fourier-transform this equation and thereby find the dielectric function $\epsilon(q)$. (1 Pt.)

(d) Assume now that $\rho^{\text{ext}}(\boldsymbol{r})$ describes a point charge placed at the origin. Calculate $\varphi(\boldsymbol{r})$. The Fourier integral can be computated by using the residue theorem. (2 Pts.)

(e) Repeat exercise (d) for one and two dimensions. (2 Pts.)