Übungen zur Theorie der Kondensierten Materie II SS 16

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1. Distribution function and susceptibility in Fermi liquid theory (9 Points)

In this exercise we will take a closer look at two aspects of Landau's Fermi liquid theory that have already been discussed in the lecture.

(a) The expression for the entropy of a Fermi liquid is given by

$$S = -k_B \sum_{\boldsymbol{k},\sigma} \left[n_{\boldsymbol{k}\sigma} \ln(n_{\boldsymbol{k}\sigma}) + (1 - n_{\boldsymbol{k}\sigma}) \ln(1 - n_{\boldsymbol{k}\sigma}) \right] \tag{1}$$

with $n_{\boldsymbol{k}\sigma}$ denoting the quasiparticle occupation number. Justify Eq. (1). Show that extremizing S at fixed particle number $N = \sum_{\boldsymbol{k},\sigma} n_{\boldsymbol{k}\sigma}$ and total energy $E = \sum_{\boldsymbol{k},\sigma} \epsilon_{\boldsymbol{k}\sigma} n_{\boldsymbol{k}\sigma}$ leads to

$$n_{\boldsymbol{k}\sigma} = \frac{1}{e^{\beta(\epsilon_{\boldsymbol{k}\sigma} - \mu)} + 1} \tag{2}$$

upon properly choosing the Lagrange multipliers. (3 Pts.)

(b) In the lecture, it has been shown that the change of the quasiparticle energies $\delta\epsilon_{\boldsymbol{k}\sigma} = t_l^\sigma Y_{lm}(\boldsymbol{e_k})$ (Y_{lm} denote spherical harmonics) resulting from a (weak) perturbation of the bare energies $\delta\epsilon^0_{\boldsymbol{k}\sigma} = v_l^\sigma Y_{lm}(\boldsymbol{e_k})$ is determined by

$$t_l^\sigma Y_{lm}(\boldsymbol{e_k}) = v_l^\sigma Y_{lm}(\boldsymbol{e_k}) - \frac{1}{N} \sum_{\boldsymbol{k}',\sigma'} f_{\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma'} \delta(\epsilon_{\boldsymbol{k}'\sigma'}) t_l^{\sigma'} Y_{lm}(\boldsymbol{e_{k'}}) \tag{3}$$

with Landau parameter $f_{\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma'}$. Using the orthogonality of the spherical harmonics derive an algebraic relation between v_l^σ and t_l^σ from Eq. (3) involving the dimensionless Landau parameters $F_l^{s,a}$ defined in the lecture. (3 Pts.)

(c) Show that the charge susceptibility of the Fermi liquid $\chi_c \propto \partial N/\partial\mu$ is related to its noninteracting limit $\chi_c^{(0)}$ via

$$\chi_c = \chi_c^{(0)} \frac{m^*/m}{1 + F_0^s} \tag{4}$$

with m and m^* denoting the bare and effective mass, respectively. (3 Pts.)

(d) *Bonus question:* Generalize Eq. (4) to higher angular momentum channels ($l \neq 0$) and spin susceptibilities.

2. Effective mass and Galilean invariance (9 Points)

Here we will derive the relation

$$\frac{m^*}{m} = 1 + \frac{1}{3} F_1^s \tag{5}$$

between the (dimensionless) spin-symmetric $l = 1$ Landau parameter F_1^s and the mass enhancement m^*/m. We will see that Eq. (5) follows from Galilean invariance. To this end, we consider an isotropic Fermi liquid at $T = 0$ in a frame moving with velocity $\boldsymbol{u}$ and calculate the quasiparticle energy $\epsilon'_{\boldsymbol{p}}$ in the moving frame in two different ways.

(a) First, convince yourself that the total energy E' of the system in the moving frame and the total momentum $\boldsymbol{P}'$ read

$$E' = E - \boldsymbol{P} \cdot \boldsymbol{u} + \frac{1}{2} M \boldsymbol{u}^2, \tag{6a}$$

$$\boldsymbol{P}' = \boldsymbol{P} - M\boldsymbol{u}, \tag{6b}$$

where E and $\boldsymbol{P}$ are the total energy and momentum in the lab frame and $M = Nm$ with N denoting the number of particles in the system. (2 Pts.)

(b) Imagine adding a quasiparticle with momentum $\boldsymbol{p}$ and corresponding energy $\epsilon_{\boldsymbol{p}}$ in the lab frame. Use Eq. (6) to show that

$$\epsilon'_{\boldsymbol{p}} \sim \epsilon_{\boldsymbol{p}} + \frac{m - m^*}{m^*} \boldsymbol{p} \cdot \boldsymbol{u}, \qquad \boldsymbol{u} \to 0, \tag{7}$$

for any $\boldsymbol{p}$ on the Fermi surface. (2 Pts.)

(c) Now let us calculate $\epsilon'_{\boldsymbol{p}}$ within the phenomenological Landau Fermi-liquid theory in order to connect with the Landau parameters. In the moving frame, the Fermi sea is shifted by $m\boldsymbol{u}$. For this reason, it holds

$$\epsilon'_{\boldsymbol{p}} = \epsilon_{\boldsymbol{p}}\big|_{n^0_{\boldsymbol{p}} \to n^0_{\boldsymbol{p}+m\boldsymbol{u}}} \tag{8}$$

with $n^0_{\boldsymbol{p}} = \theta(-\epsilon_{\boldsymbol{p}})$ denoting the Fermi-distribution function at $T = 0$. By expanding Eq. (8) in $\boldsymbol{u}$, show that

$$\epsilon'_{\boldsymbol{p}} \sim \epsilon_{\boldsymbol{p}} - \frac{1}{N} \frac{m}{m^*} \sum_{\boldsymbol{p}', \sigma'} \delta(\epsilon_{\boldsymbol{p}'}) f_{\boldsymbol{p}\sigma, \boldsymbol{p}'\sigma'} \, \boldsymbol{p}' \cdot \boldsymbol{u}, \qquad \boldsymbol{u} \to 0, \tag{9}$$

with the phenomenological Landau parameter $f_{\boldsymbol{k}\sigma, \boldsymbol{k}'\sigma'}$ introduced in the lecture. (2 Pts.)

(d) Compare Eqs. (7) and (9) to show Eq. (5). (3 Pts.)