Karlsruher Institut für Technologie

Institut für Theorie
der Kondensierten Materie

Theorie der Kondensierten Materie II   SS 2017

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1. Normal ordering in 1D: (60 + 40 = 100 Punkte)

As discussed in the lecture, one builds the effective low energy theory in 1D using the concept of normal ordering. This way, one may separate singular terms in a product of operators.

(a) In the lecture, we have considered in detail normal ordering of two fermionic field operators. The product of four ferminc field operators is given by

$$\psi_R^\dagger(x_1)\psi_R(x_2)\psi_R^\dagger(x_3)\psi_R(x_4) =: \psi_R^\dagger(x_1)\psi_R(x_2)\psi_R^\dagger(x_3)\psi_R(x_4) :$$

$$+\langle\psi_R^\dagger(x_1)\psi_R(x_2)\rangle : \psi_R^\dagger(x_3)\psi_R(x_4) : +\langle\psi_R^\dagger(x_1)\psi_R(x_4)\rangle : \psi_R(x_2)\psi_R^\dagger(x_3) :$$

$$+\langle\psi_R^\dagger(x_3)\psi_R(x_4)\rangle : \psi_R^\dagger(x_1)\psi_R(x_2) : +\langle\psi_R(x_2)\psi_R^\dagger(x_3)\rangle : \psi_R^\dagger(x_1)\psi_R(x_4) :$$

$$+\langle\psi_R^\dagger(x_1)\psi_R(x_4)\rangle\langle\psi_R(x_2)\psi_R^\dagger(x_3)\rangle + \langle\psi_R^\dagger(x_1)\psi_R(x_2)\rangle\langle\psi_R^\dagger(x_3)\psi_R(x_4)\rangle$$

Derive this relation using the methods shown in the lecture.

(b) Consider now the product of two normal ordered products, given by the relation

$$: \psi_R^\dagger(x_1)\psi_R(x_2) :: \psi_R^\dagger(x_1)\psi_R(x_4) :=: \psi_R^\dagger(x_1)\psi_R(x_2)\psi_R^\dagger(x_3)\psi_R(x_4) :$$

$$+ : \psi_R^\dagger(x_1)\psi_R(x_4) : \langle\psi_R(x_2)\psi_R^\dagger(x_3)\rangle + : \psi_R(x_2)\psi_R^\dagger(x_3) : \langle\psi_R^\dagger(x_1)\psi_R(x_4)\rangle$$

$$+\langle\psi_R^\dagger(x_1)\psi_R(x_4)\rangle\langle\psi_R(x_2)\psi_R^\dagger(x_3)\rangle$$

Derive this relation using the methods shown in the lecture.