Karlsruher Institut für Technologie
Institut für Theorie der Kondensierten Materie

Theorie der Kondensierten Materie II SS 2017

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1. Specific heat in 1D: (15 Punkte)

Find the specific heat of a free electron gas in 1D. Calculate the specific heat using the canonical aproach and compare to the result obtained within the bosonic representation.

2. Free Fermi gas on a finite-size ring: (15 + 15 + 15 = 45 Punkte)

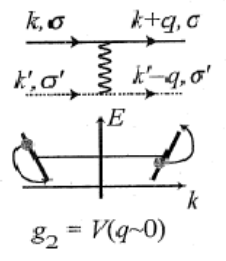
Consider a free Fermi gas in the finite region $0 < x < L$ with periodic boundary conditions.

(a) Find the discrete signle-particle energy level. Explain how would you construct many-body states. Show that they will also be discrete.

(b) Linearize the spectrum. Show, that the discrete spectrum of many-body states has the form $E_m = m\Delta$, with $\Delta = 2\pi v_F/L$. Find the degeneracy N_m for a few lowest levels.

(c) Repeat the above considertions within the bosonic representation. Show, that this leads to identical results.

3. Tunneling density of states (40 Punkte)



Consider now a point contact between to Luttinger liquids. Each liquid is characterized by the interaction terms

$$g_2^{A(B)} R^\dagger_{k+q,\sigma} R_{k,\sigma} L^\dagger_{k'-q,\sigma'} L_{k'\sigma'}.$$

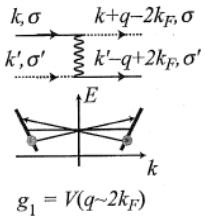
Tunneling through the point contact can be described by the Hamiltonian

$$\hat{H}_{tun} = t\psi^\dagger_B(x=0)\psi_A(x=0) + h.c.$$

Show that at $T = 0$ the tunneling current is given by a power law $I \propto V^{\alpha}$ and relate the exponent to the coupling constants.

** *Extra exercise*

(40 Punkte)



What would happen to your results if you take into account g_1 interaction processes as well?

$$g_1 L^{\dagger}_{k+q-2k_F,\sigma} L_{k',\sigma'} R^{\dagger}_{k'-q+2k_F,\sigma'} R_{k,\sigma}.$$