Karlsruher Institut für Technologie

Institut für Theorie der Kondensierten Materie

Übungen zur Theorie der Kondensierten Materie II SS 18

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Blatt 9

Besprechung 29.06.2016

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1. Matsubara summation II (15 + 15 + 20 Points)

In part 2(b) of the previous problem sheet you showed that the sum

$$S(t) = T \sum_{n \in \mathbb{Z}} f(i\omega_n) e^{i\omega_n t}, \tag{1}$$

can be written as a contour integral

$$S(t) = \frac{-\eta}{2\pi i} \oint_{\mathcal{C}} dz\; n_\eta(z) f(z) e^{zt}, \tag{2}$$

where $\mathcal{C}$ consists of two vertical lines enclosing the infinite number of poles of n_η.

(a) Perform the Matsubara summation in the expression

$$\Sigma(\omega_m) = T \sum_{i\omega_n} D(\omega_n)\, G(\omega_m + \omega_n)\,. \tag{3}$$

Here, $D(\omega_n, \boldsymbol{q})$ is the bosonic propagator and $G(\omega_m, \boldsymbol{k})$ is the fermion propagator. ω_n are bosonic Matsubara frequencies.

(b) Assume now that $f(z)$ is analytic everywhere except on the real axis. By deforming $\mathcal{C}$ show that

$$S = \frac{-\eta}{2\pi i} \int_{-\infty}^{\infty} d\omega\; n_\eta(\omega) \left[f(\omega + i0^+) - f(\omega - i0^+)\right] e^{\omega t} \tag{4}$$

holds. You may assume that $f(z)$ decays in a suitable form as $z \to \infty$.

(c) For a non-interacting electron gas, the free energy can be written in terms of the Matsubara Green's function

$$F = -T \sum_{\boldsymbol{k}} \sum_{n} \log\left[-\mathcal{G}^{-1}_{0,\boldsymbol{k}}(\omega_n)\right] e^{i\omega_n 0^+}. \tag{5}$$

Using part (a), compute the sum over n and show that F is the well-known expression from non-interacting fermion theory. In order to carry out the calculation, you will have to use the fact that the logarithm can only be defined with a branch-cut (if one wants to avoid multivalued functions). It is convenient to choose a definition where the logarithm $\log z$ has its branch-cut in $[-\infty, 0]$.