Moderne Theoretische Physik IIIb (Theorie Fb) Sommersemester 2019

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1. Langevin-Gleichung: (10+25=35 Punkte)

Betrachten Sie einen stromgetriebenen parallelen LRC-Schwingkreis. Das Verhältnis zwischen Spannung $V(t)$ und Strom $I(t)$ ergibt sich aus der Bewegungsgleichung

$$C\ddot{V} + \frac{\dot{V}}{R} + \frac{V}{L} = \dot{I}, \tag{1}$$

wobei $I(t) = I_0(t) + \delta I(t)$ mit $\langle I(t)\rangle = I_0(t)$ und δI sogenanntes Nyquist-Rauschen des Stroms durch den Widerstand beschreibt. Es gilt:

$$\langle \delta I(t)\delta I(t')\rangle = \frac{2k_B T}{R}\delta(t-t'). \tag{2}$$

(a) Bestimmen Sie die Impedanz $Z(\omega) = V(\omega)/I(\omega)$ durch Fouriertransformation der Bewegungsgleichung.

Solution:

By using the Fourier convention (where X is any function of time)

$$X(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega t} X(\omega) \Leftrightarrow X(\omega) = \int_{-\infty}^{\infty} dt\, e^{-i\omega t} X(t), \tag{3}$$

Eq. (1) transforms into

$$V(\omega)\left[i\omega C + \frac{1}{R} + \frac{1}{i\omega L}\right] = I(\omega). \tag{4}$$

The impedance then reads

$$Z(\omega) \equiv \frac{V(\omega)}{I(\omega)} = \frac{1}{i\omega C + \frac{1}{R} + \frac{1}{i\omega L}} = \frac{i\omega}{-\omega^2 C + \frac{i\omega}{R} + \frac{1}{L}}, \tag{5}$$

where the last equality will be used for later purposes. □

(b) Berechnen Sie die Korrelationen des Spannungsrauschens $\langle \delta V(t)\delta V(t')\rangle$ im Fall $(2RC)^2 > LC$.
Hinweis: Berechnen Sie zunächst $\langle \delta V(\omega)\delta V(\omega')\rangle$. Bei der Rücktransformation vom Frequenz- in den Zeitraum ist der Residuensatz nützlich.

Solution:

For any quantity X, the fluctuations are defined as

$$\delta X = X - \langle X\rangle. \tag{6}$$

For any two quantities, the following identity holds

$$\langle \delta X \delta Y \rangle = \langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle = \langle XY \rangle - \langle X \rangle \langle Y \rangle. \tag{7}$$

We then have that

$$\begin{aligned}
&\langle \delta V(\omega) \delta V(\omega') \rangle = \langle V(\omega) V(\omega') \rangle - \langle V(\omega) \rangle \langle V(\omega') \rangle = \\
&\langle Z(\omega) I(\omega) Z(\omega') I(\omega') \rangle - \langle Z(\omega) I(\omega) \rangle \langle I(\omega') Z(\omega') \rangle = \\
&Z(\omega) Z(\omega') \left[\langle I(\omega) I(\omega') \rangle - \langle I(\omega) \rangle \langle I(\omega') \rangle \right] = Z(\omega) Z(\omega') \langle \delta I(\omega) \delta I(\omega') \rangle.
\end{aligned} \tag{8}$$

We next compute

$$\langle \delta I(\omega) \delta I(\omega') \rangle = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \, e^{-i\omega t} e^{-i\omega' t'} \langle \delta I(t) \delta I(t') \rangle = \frac{4\pi k_B T}{R} \delta(\omega + \omega'), \tag{9}$$

where we in the last step used Eq. (2) as well as the identity

$$\int_{-\infty}^{\infty} dt \, e^{-i(\omega + \omega')t} = 2\pi \delta(\omega + \omega'). \tag{10}$$

We therefore obtain

$$\langle \delta V(\omega) \delta V(\omega') \rangle = \frac{4\pi k_B T}{R} Z(\omega) Z(\omega') \delta(\omega + \omega'). \tag{11}$$

We next Fourier transform back to the time domain

$$\begin{aligned}
&\langle \delta V(t) \delta V(t') \rangle = \langle \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega t} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} e^{i\omega' t'} \delta V(\omega) \delta V(\omega') \rangle = \\
&\frac{2k_B T}{R} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} Z(\omega) Z(-\omega) e^{-i\omega(t-t')}.
\end{aligned} \tag{12}$$

Next, we use Eq. (5) to compute

$$Z(\omega) Z(-\omega) = \frac{\omega^2}{(-\omega^2 C + \frac{i\omega}{R} + \frac{1}{L})(-\omega^2 C - \frac{i\omega}{R} + \frac{1}{L})}. \tag{13}$$

The poles of this function are located at

$$\omega_1 = \frac{1}{\alpha}(i + \beta), \tag{14}$$

$$\omega_2 = \frac{1}{\alpha}(i - \beta), \tag{15}$$

$$\omega_3 = \frac{1}{\alpha}(-i + \beta), \tag{16}$$

$$\omega_4 = \frac{1}{\alpha}(-i - \beta), \tag{17}$$

where we have defined $\alpha \equiv 2RC$, and $\beta = \sqrt{4CR^2/L - 1}$. Since $(2RC)^2 > LC$, we have that $\beta > 0$. What remains now is to perform the integral in Eq. (12). To this end, we define $\tau = t - t'$. Let us first assume that $\tau > 0$. We then extend the integral over ω to the complex plane and choose a contour γ that lies along the real axis and closes by an arc in the lower complex half plane which encircles the poles ω_3 and ω_4 (since $\tau > 0$ by assumption, the vanishing of the arced part holds only with this choice). By the residue theorem, we obtain

$$\langle\delta V(t)\delta V(t')\rangle = \frac{2k_BT}{R}\oint_\gamma \frac{d\omega}{2\pi} Z(\omega)Z(-\omega)e^{-i\omega\tau} =$$
$$\frac{2k_BT}{C^2R}\frac{2\pi(-i)}{2\pi}\left[\frac{e^{-i\omega_3\tau}\omega_3^2}{(\omega_3-\omega_1)(\omega_3-\omega_2)(\omega_3-\omega_4)} + \frac{e^{-i\omega_4\tau}\omega_4^2}{(\omega_4-\omega_1)(\omega_4-\omega_2)(\omega_4-\omega_3)}\right] =$$
$$\frac{k_BT\alpha}{2RC^2\beta}e^{-\tau/\alpha}\left(\beta\cos(\frac{\beta\tau}{\alpha}) - \sin(\frac{\beta\tau}{\alpha})\right). \tag{18}$$

We next assume $\tau < 0$ and close the γ contour in the upper half plane. The contour then encircles ω_1 and ω_2 and we may again use the residue theorem. This yields the same expression as Eq. (18) but with $t \to -t$. We may therefore replace τ with $|\tau|$ and obtain our final result

$$\langle\delta V(t)\delta V(t')\rangle = \frac{k_BT}{C}e^{-|\tau|/\alpha}\left(\cos(\omega_c|\tau|) - \frac{\sin(\omega_c|\tau|)}{\alpha\omega_c}\right), \tag{19}$$

which is an exponentially enveloped oscillating function in $|\tau|$. We have here defined the characteristic oscillation frequency

$$\omega_c \equiv \frac{\beta}{\alpha} = \sqrt{\frac{1}{LC} - \frac{1}{4C^2R^2}} = \sqrt{\omega_0^2 - \alpha^{-2}}. \tag{20}$$

which reflects the competition between the LC frequency $\omega_0 = \sqrt{1/LC}$ and the characteristic decay (attenuation) time α. The expression (19) manifests a fluctuation-disspiation relation: the voltage fluctuation correlations depends on the dissipative quantity α. □

2. Verzögerte Dämpfung: (10+15+25+15=65 Punkte)

Dissipation lässt sich auch quantenmechanisch modellieren (A. Caldeira und A. Leggett, 1981). Das führt schließlich auf die klassische Bewegungsgleichung

$$m\ddot{x}(t) + m\int_0^t ds\ K(t-s)\dot{x}(s) = F(t), \tag{21}$$

wobei $F(t)$ eine gegebene Kraft ist. Hierbei wird die Dämpfung durch den Integralkern

$$K(t) = \Theta(t)\gamma_0\omega_d e^{-\omega_d t}$$

beschrieben, wobei $\Theta(t)$ die Heaviside-Theta-Funktion ist. Die Anfangsbedingungen sind $x(0) = x_0$ und $\dot{x}(0) = v_0$.

(a) Finden Sie $x(t)$ im Limes $\omega_d \to \infty$.

Solution:

In this limit, we see that $K(t-s) \to \gamma_0\delta(t-s)$ and the equation of motion is reduced to

$$m\ddot{x}(t) + m\gamma_0\dot{x}(t) = F(t). \tag{22}$$

Since $\dot{x}(t) = v(t)$, the equation for the velocity reads

$$m\dot{v}(t) + m\gamma_0 v(t) = F(t), \tag{23}$$

which has the typical solution of first order linear non-homogenous differential equations:

$$v(t) = v_0 e^{-\gamma_0 t} + \int_0^t ds\; e^{-\gamma_0(t-s)} \frac{F(s)}{m}. \tag{24}$$

We can perform one more integration to finally obtain $x(t)$ [1]

$$x(t) = x_0 + \frac{v_0}{\gamma_0}(1 - e^{-\gamma_0 t}) + \int_0^t ds\; \left[1 - e^{-\gamma_0(t-s)}\right] \frac{F(s)}{\gamma_0 m}. \;\square \tag{25}$$

(b) Bestimmen Sie die Bewegungsgleichung für die Laplace-Transformierte von $x(t)$,

$$\tilde{x}(z) = \int_0^\infty dt\; x(t) e^{-zt}.$$

Solution:

From the definition of the Laplace transform, one may derive the following rules

$$\ddot{x}(t) \to z^2 \tilde{x}(z) - z x_0 - v_0, \tag{26}$$

$$\dot{x}(t) \to z\tilde{x}(z) - x_0, \tag{27}$$

$$\int_0^t ds \dot{x}(s) K(t-s) \to z\tilde{x}(z)\tilde{K}(z). \tag{28}$$

The memory kernel transforms into

$$\tilde{K}(z) = \int_0^\infty dt e^{-zt} \Theta(t) \gamma_0 \omega_d e^{-\omega_d t} = \int_0^\infty dt \gamma_0 \omega_d e^{t(-\omega_d - z)} = \frac{\gamma_0 \omega_d}{z + \omega_d}. \tag{29}$$

Using the above, Eq. (22) transforms into

$$z^2 \tilde{x}(z) + \frac{z\tilde{x}(z)\gamma_0\omega_d}{z + \omega_d} - z x_0 - v_0 - \frac{x_0 \gamma_0 \omega_d}{z + \omega_d} = \frac{\tilde{F}(z)}{m} \tag{30}$$

$$\Leftrightarrow \tilde{x}(z)\left(z^2 + \frac{z\gamma_0\omega_d}{z + \omega_d}\right) - z x_0 - v_0 - \frac{x_0 \gamma_0 \omega_d}{z + \omega_d} = \frac{\tilde{F}(z)}{m}. \;\square \tag{31}$$

(c) Die Suszeptibilität $\tilde{\chi}(z)$ wird durch die Relation

$$\tilde{x}_0(z) = \tilde{\chi}(z)\tilde{F}(z)$$

definiert, wobei $\tilde{x}_0(z)$ der von den Anfangsbedingungen unabhängige Teil von $\tilde{x}(z)$ ist. Zeigen Sie, dass

$$\tilde{\chi}(z) = \frac{1}{m} \frac{z + \omega_d}{z(z^2 + z\omega_d + \gamma_0\omega_d)}.$$

[1] To construct and verify such solutions, it is useful to apply Leibniz integral rule: $\frac{d}{dt}(\int_0^t f(s,t)ds) = f(t,t) + \int_0^t \frac{\partial}{\partial t}(f(s,t))ds$

Finden Sie die inverse Laplace-Transformation der Suszeptibilität

$$\chi(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dz\, e^{zt}\tilde{\chi}(z), \qquad t > 0, \quad c > 0,$$

unter Verwendung von Konturintegralen in der komplexen Ebene.

Solution:

Ignoring the boundary parts we find

$$\tilde{\chi}(z) \equiv \frac{\tilde{x}_0(z)}{\tilde{F}(z)} = \frac{1}{m}\frac{z+\omega_d}{z(z^2+z\omega_d+\gamma_0\omega_d)}. \tag{32}$$

To transform back, we write

$$\chi(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dz\, e^{zt}\frac{1}{m}\frac{z+\omega_d}{z(z-z_1)(z-z_2)}, \tag{33}$$

where the poles are located at $z = 0$ and $z_{1/2} = (-\omega_d \pm \omega_c)/2$ with $\omega_c \equiv \sqrt{\omega_d(\omega_d - 4\gamma_0)}$. We note that the real part of $z_{1/2}$ is always negative. ω_c can be either real or complex. To perform the integral, we extend the integration contour to form an arc in the left half-plane which then encircles all poles counter clockwise (since $c > 0$). We can then use the residue theorem and we obtain

$$\chi(t) = \frac{1}{2\pi i m}2\pi i\left(\frac{e^{z_1 t}(z_1+\omega_d)}{z_1(z_1-z_2)} + \frac{e^{z_2 t}(z_2+\omega_d)}{z_2(z_2-z_1)} + \frac{\omega_d}{z_1 z_2}\right). \tag{34}$$

Inserting the poles, we obtain our final result as

$$\begin{aligned} m\chi(t) = &-\frac{4\omega_d}{\omega_d^2-\omega_c^2} + \frac{(\omega_c+\omega_d)(\cosh(\frac{t(\omega_c-\omega_d)}{2}) + \sinh(\frac{t(\omega_c-\omega_d)}{2})}{\omega_c(\omega_c-\omega_d)} \\ &+ \frac{(\omega_c-\omega_d)(\cosh(\frac{t(\omega_c+\omega_d)}{2}) - \sinh(\frac{t(\omega_c+\omega_d)}{2})}{\omega_c(\omega_c+\omega_d)}. \;\square \end{aligned} \tag{35}$$

(d) Betrachten wir nun eine konstante Kraft $F(t) = F_0$ für $t > 0$. Finden Sie das Verhalten von $x(t)$ im Limes $t \to \infty$.

Solution:

The Laplace transform of the constant force becomes

$$\tilde{F}(z) = \int_0^\infty dt\; F_0\Theta(t)e^{-zt} = \frac{F_0 e^{-zt}}{-z}|_0^\infty = \frac{F_0}{z}. \tag{36}$$

If we use this result in Eq. (30) together with the definition of $\tilde{\chi}(z)$, we obtain

$$\tilde{x}(z) = \left(zx_0 + v_0 + \frac{x_0\gamma_0\omega_d}{z+\omega_d}\right)m\tilde{\chi}(z) + \tilde{\chi}(z)\frac{F_0}{z}. \tag{37}$$

To find $x(t \to \infty)$, we could transform this equation back to the time domain and perform the limit. It is however much more convenient to use the initial and final value theorems of the Laplace transform

$$x(0^-) = \lim_{z\to\infty} z\tilde{x}(z) = x_0 \tag{38}$$
$$x(\infty) = \lim_{z\to 0} z\tilde{x}(z) = \infty \tag{39}$$
$$v(0^-) = \lim_{z\to\infty} z(z\tilde{x}(z) - x_0) = v_0 \tag{40}$$
$$v(\infty) = \lim_{z\to 0} z(z\tilde{x}(z) - x_0) = \frac{F_0}{m\gamma_0}. \tag{41}$$

We conclude that as $t \to \infty$ the "particle" approaches infinity with a "terminal velocity" $v_f = \frac{F_0}{m\gamma_0}$. $\square$

3. Boltzmann-Gleichung: (25 Bonuspunkte)

Betrachten Sie ein zweidimensionales Elektronengas mit isotroper Dispersion in der xy-Ebene im externen Magnetfeld $\vec{B} = B\vec{e}_z$. Ausgehend von der Boltzmann-Gleichung bestimmen Sie den Leitfähigkeitstensor $\sigma_{\alpha\beta}(\omega)$. Es sei angenommen, dass das Stoßintegral die isotrope elastische Streuung von Verunreinigungen beschreibt, d.h. die Wahrscheinlichkeit der Elektronenstreuung $\vec{k} \to \vec{k}'$ pro Zeiteinheit ist unabhängig von $\vec{k}$ und $\vec{k}'$, $W_{\vec{k}\vec{k}'} = W_0$.

Solution:

The Boltzmann equation for the distribution function $f(\vec{r}, \vec{k}, t) = f_0(\vec{r}, \vec{k}, t) + \delta f(\vec{r}, \vec{k}, t)$ reads (for notational ease, we often suppress the variable dependencies)

$$\partial_t f + \vec{v} \cdot \nabla_{\vec{r}} f - \frac{e}{\hbar}\left[\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right] \cdot \nabla_{\vec{k}} f = I[f], \tag{42}$$

where $\vec{B} = B\hat{z}$ and the spatially constant but time dependent external electrical field reads $\vec{E}(w) = \vec{E}_\omega e^{-i\omega t}$. We are interested in the linear response regime and therefore assume that δf is both linear in $\vec{E}_\omega$ as well as spatially constant, and also has the same time dependence $\delta f = \delta f_\omega e^{-i\omega t}$. In this regime, we therefore have $\vec{E} f \approx \vec{E} f_0$.

The collision integral reads

$$I[f] = -W_0 \int \frac{d^2k'}{(2\pi)^2}(f(\vec{k}) - f(\vec{k}')), \tag{43}$$

where we next assume elastic collisions. It then follows that the isotropic dispersion relation $\epsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m}$ requires $|\vec{k}| = |\vec{k}'|$. Furthermore, we have that $f_0(\vec{k}) = f_0(|\vec{k}|)$ for f_0 being the Fermi distribution function. We then have that $f(\vec{k}) - f(\vec{k}') = \delta f(\vec{k}) - \delta f(\vec{k}')$. Next, we assume that δf is linear[2] in $\vec{k}$. The integral over $\delta f(\vec{k}')$ then vanishes (we integrate an odd function over an isotropic volume) and the collision integral becomes

$$I[f] = -W_0 \int \frac{d^2k'}{(2\pi)^2}(f(\vec{k}) - f(\vec{k}')) = -W_0 \delta f(\vec{k}) \int \frac{d^2k'}{(2\pi)^2} \equiv -\frac{\delta f(\vec{k})}{\tau}, \tag{44}$$

where we defined the elastic scattering time $\tau \equiv W_0 \Omega$ with Ω being the phase space volume.

[2] In short, this can be justified on the grounds of assuming a spatially uniform system.

We further note that

$$\nabla_{\vec{k}} f_0 = \frac{\partial f_0}{\partial \epsilon_{\vec{k}}} \frac{\partial \epsilon_{\vec{k}}}{\partial \vec{k}} = -\frac{\partial f_0}{\partial \mu} \hbar \vec{v}(\vec{k}), \tag{45}$$

where we have used , and that $f_0 = f_0(\epsilon_{\vec{k}} - \mu)$. The velocity $\vec{v}(\vec{k}) = \frac{\hbar \vec{k}}{m}$. It follows that

$$(\vec{v}(\vec{k}) \times \vec{B}) \cdot \nabla_{\vec{k}} f_0 = 0. \tag{46}$$

The linearized Boltzmann equation for δf_ω then reads

$$-e\vec{E} \cdot \vec{v}(\vec{k})(-\frac{\partial f_0}{\partial \mu}) - \frac{e}{\hbar c}(\vec{v}(\vec{k}) \times \vec{B}) \cdot \partial_{\vec{k}} \delta f_\omega(\vec{k}) = -\frac{1}{\tau_\omega} \delta f_\omega(\vec{k}), \tag{47}$$

where we have defined $\frac{1}{\tau_\omega} = (\frac{1}{\tau} - i\omega)$. We next make the ansatz $\delta f_\omega = ak_x + bk_y$ and plug it into the linearized Boltzmann equation. Matching coefficients for k_x and k_y (which are independent variables) leads to

$$a = \frac{E_{\omega,x} - \omega_c \tau_\omega E_{\omega,y}}{1 + (\omega_c \tau_\omega)^2} (\frac{-e\hbar \tau_\omega}{m})(\frac{\partial f_0(\vec{k})}{\partial \mu}), \tag{48}$$

$$b = \frac{E_{\omega,y} + \omega_c \tau_\omega E_{\omega,x}}{1 + (\omega_c \tau_\omega)^2} (\frac{-e\hbar \tau_\omega}{m})(\frac{\partial f_0(\vec{k})}{\partial \mu}), \tag{49}$$

where we have defined the cyclotron frequency $\omega_c = \frac{eB}{mc}$.
The current densities j_α (with $\alpha = x, y$) read

$$j_\alpha = -2e \int \frac{d^2k}{(2\pi)^2} v_\alpha(\vec{k})(ak_x + bk_y) = -\frac{2em}{\hbar} \int \frac{d^2k}{(2\pi)^2} (av_\alpha(k)^2 \delta_{\alpha x} + bv_\alpha(k)^2 \delta_{\alpha y}), \tag{50}$$

where we have used that $\int \frac{d^2k}{(2\pi)^2} av_\alpha(\vec{k}) = \int \frac{d^2k}{(2\pi)^2} bv_\alpha(\vec{k}) = 0$ due to the isotropy.
Plugging in the expressions for a and b, we find the contributions

$$\begin{aligned}
&-\frac{2em}{\hbar} \int \frac{d^2k}{(2\pi)^2} v_\alpha(k)^2 (\frac{-e\hbar\tau_\omega}{m})(\frac{\partial f_0(\vec{k})}{\partial \mu}) \\
&\approx \frac{2e^2\hbar^2\tau_\omega}{m^2} \int_0^\infty \frac{dk}{(2\pi)^2} \int_0^{2\pi} d\varphi k k_\alpha^2 \delta(\epsilon_{\vec{k}} - \mu) \\
&= 2e^2\tau_\omega \frac{\hbar^2}{4\pi m^2} \int_0^\infty dk k^3 m \frac{\delta(k - k_F)}{\hbar^2 k_F} \\
&= 2e^2\tau_\omega \frac{k_F^2}{4\pi m} = \frac{k_F^2}{2\pi} \frac{e^2\tau_\omega}{m} = \frac{ne^2\tau_\omega}{m} \equiv \sigma_0,
\end{aligned} \tag{51}$$

where in the first step, we used $(\frac{\partial f_0(\vec{k})}{\partial \mu}) \approx \delta(\epsilon_{\vec{k}} - \mu)$, valid for f_0 being the Fermi distribution function and for low temperatures $k_B T \ll \mu$. We also used $k_x^2 = k^2 \cos^2 \varphi$, $k_y^2 = k^2 \sin^2 \varphi$ and that $\int_0^{2\pi} \cos^2 \varphi = \int_0^{2\pi} \sin^2 \varphi = \pi$. In $2D$, the electron density $n = \frac{k_F^2}{2\pi}$.
We then obtain for the current densities

$$j_\alpha = \frac{\sigma_0}{1 + (\omega_c\tau_\omega)^2} \left(\delta_{\alpha x}(E_{\omega,x} - \omega_c\tau_\omega E_{\omega,y}) + \delta_{\alpha y}(E_{\omega,y} + \omega_c\tau_\omega E_{\omega,x})\right). \tag{52}$$

After some rearranging, we finally obtain the conductivity tensor

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \frac{\sigma_0}{1 + (\omega_c\tau_\omega)^2} \begin{pmatrix} 1 & -\omega_c\tau_\omega \\ \omega_c\tau_\omega & 1 \end{pmatrix} \begin{pmatrix} E_{\omega,x} \\ E_{\omega,y} \end{pmatrix} \equiv \sigma(\omega) \begin{pmatrix} E_{\omega,x} \\ E_{\omega,y} \end{pmatrix}. \tag{53}$$

Again, $\omega_c = \frac{eB}{mc}$ is the cyclotron frequency, and $\tau_\omega = (1/\tau - i\omega)^{-1}$ is the frequency dependent scattering time. Inverting $\sigma(\omega)$, we find the resistivity tensor

$$\rho(\omega) = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c \tau_\omega \\ -\omega_c \tau_\omega & 1 \end{pmatrix} = \begin{pmatrix} \frac{m}{ne^2 \tau_\omega} & \frac{B}{nec} \\ -\frac{B}{nec} & \frac{m}{ne^2 \tau_\omega} \end{pmatrix} \tag{54}$$

The off-diagonal elements $\rho_{xy} = -\rho_{yx} = \frac{B}{nec}$, are independent of the intrinsic scattering rate τ and the driving frequency ω of the electrical field. We further note that the real part of the longitudinal conductivity $\sigma_{xx} = \sigma_{yy}$ has a resonance at $\omega = \omega_c$. At this frequency, the conductivity in enhanced. □

Bitte melden Sie sich sowohl zur Vorleistung 1 (Tutorien) als auch zur 1. Klausur im Campus-System an!