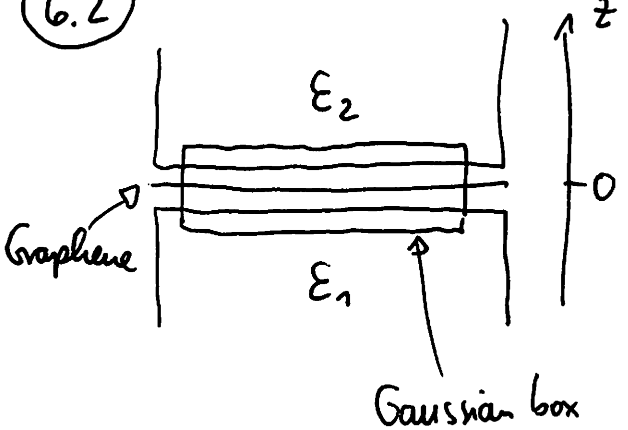
6.2



Maxwell eqn. free charge

$$\nabla \cdot \vec{D} = 4\pi \rho_f \Rightarrow D_{1,n} - D_{2,n} = 4\pi\,\sigma(x,y)$$

$$\vec{D} = \varepsilon \vec{E}.$$

charge area density in boundary layer graphene.

and $\nabla\phi = -\vec{E}$ for electric potential ϕ.

(a) assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$.

$$\Rightarrow \nabla \cdot \vec{D} = 4\pi \rho_f$$

$$\Leftrightarrow \nabla \cdot (\varepsilon \vec{E}) = 4\pi \rho_f \Leftrightarrow \varepsilon \nabla \cdot \vec{E} = 4\pi \rho_f \Leftrightarrow \boxed{-\varepsilon \nabla^2 \phi = 4\pi \rho_f.}$$ (Poisson eq.

now assume $\rho_f = \sigma(x,y)\,\delta(z)$ (e.g. $\sigma(x,y) = -e_0\,\delta(\vec{x} - \frac{\vec{r}}{2}) - e_0\,\delta(\vec{x} + \frac{\vec{r}}{2})$).

Go to Fourier space.

$$\phi(\vec{x}) = \int_q \phi(\vec{q})\, e^{i\vec{q}\cdot\vec{x}} \Rightarrow +\varepsilon\left(\vec{q}_\perp^{\,2} + q_z^2\right)\phi_{\vec{q}} = 4\pi\,\sigma(q_\perp)$$

$$\rho_f(\vec{q}_\perp, q_z) = \int dx\,dy\,dz\; \sigma(x,y)\,\delta(z)\, e^{-i\vec{q}\cdot\vec{x}} = \sigma(\vec{q}_\perp)$$

$$\Rightarrow \boxed{\phi(\vec{q}_\perp, q_z) = \frac{4\pi\,\sigma(\vec{q}_\perp)}{\varepsilon(q_\perp^2 + q_z^2)}}.$$

at $z=0$: $$\phi(q_\perp, z=0) = \int_{-\infty}^{\infty} \frac{dq_z}{2\pi}\, \phi(q_\perp, q_z) = \frac{4\pi^2\sigma}{\varepsilon|\vec{q}_\perp|\cdot 2\pi} = \frac{2\pi\sigma}{\varepsilon|\vec{q}_\perp|}$$

$$\boxed{\Rightarrow \phi(q_\perp, z=0) = \frac{2\pi\,\sigma(q_\perp)}{\varepsilon|\vec{q}_\perp|}}$$

$$\cancel{|\nabla_k \varepsilon(\vec{k})|} = 0$$

$$\nabla_k \varepsilon(\vec{k}) = \vec{0}$$

(b) now: ε_1 for $z<0$ $\qquad \nabla\cdot\vec{D} = 4\pi\rho_f$

ε_2 for $z>0$.

$$\nabla(\varepsilon\vec{E}) = 4\pi\rho_f \Leftrightarrow \nabla(\varepsilon(z)\vec{E}) = 4\pi\sigma(x,y)\,\delta(z)$$

$$\Leftrightarrow \nabla\cdot(\varepsilon(z)[-\nabla\phi]) = -\nabla\cdot(\varepsilon(z)\nabla\phi) = 4\pi\sigma\,\delta(z)$$

$z>0$: $\varepsilon_2\left(\frac{d^2}{dz^2}+\nabla_\perp^2\right)\phi = -4\pi\sigma\,\delta(z)$

$z<0$: $\varepsilon_1\left(\frac{d^2}{dz^2}+\nabla_\perp^2\right)\phi = -4\pi\sigma\,\delta(z)$

$$\Downarrow$$

$$\boxed{\phi(q_\perp,z) = A_1 e^{+q_\perp z} \text{ for } z<0.}$$

$$\phi(x,y,z) = \int_{q_\perp}\phi(q_\perp,z)\,e^{iq_\perp\vec{x}} \qquad (x,y)=\vec{x}.$$

$$\Downarrow$$

$$\varepsilon_2\left(\frac{d^2}{dz^2}-q_\perp^2\right)\phi(q_\perp,z) = 4\pi\sigma\,\delta(z) \underset{z>0.}{=} 0$$

$$\Downarrow$$

$$\boxed{\phi(q_\perp,z) = A_2 e^{-q_\perp z} \quad \text{for } z>0}$$

(B.c. that $\phi(q_\perp, z\to\pm\infty) = 0$.).

ϕ is continuous at $z=0$: $\Rightarrow \boxed{A_1 = A_2}$, but $D_{1,n} - D_{2,n} = 4\pi\sigma$

$\Leftrightarrow \quad D_{1,n} - D_{2,n} = 4\pi\sigma$

$\Leftrightarrow \quad \varepsilon_1 E_{1,n} - \varepsilon_2 E_{2,n} = 4\pi\sigma$

$\Rightarrow \boxed{\phi(q_\perp, z) = A\, e^{-q_\perp |z|}} \; \forall z.$

Now find A:

$E_z(q_\perp, z) = -\frac{d}{dz}\phi(q_\perp, z)$ since $\vec{E}(\vec{x}) = -\nabla\phi = \begin{pmatrix} -\partial_x\phi \\ -\partial_y\phi \\ -\partial_z\phi \end{pmatrix}$

$$\Rightarrow \quad \varepsilon_1\left(-\frac{d}{dz}\phi\Big|_{\substack{z<0\\ z\to 0}}\right) - \varepsilon_2\left(-\frac{d}{dz}\phi\Big|_{\substack{z>0\\ z\to 0}}\right) = 4\pi\sigma$$

$$\Leftrightarrow \quad -\varepsilon_1 q_\perp A e^{-q_\perp|z|}\Big|_{\substack{z<0\\ z\to 0}} - \varepsilon_2 q_\perp A e^{-q_\perp|z|}\Big|_{\substack{z>0\\ z\to 0}} = 4\pi\sigma$$

$$\Rightarrow -(\varepsilon_1 + \varepsilon_2) q_\perp A = 4\pi\sigma$$

$$\Leftrightarrow \boxed{A = -\frac{4\pi\sigma}{q_\perp(\varepsilon_1+\varepsilon_2)}} \quad \Rightarrow \boxed{\phi(q_\perp, z) = -\frac{4\pi\sigma(q_\perp)\, e^{-q_\perp|z|}}{(\varepsilon_1+\varepsilon_2)\, q_\perp}.}$$

Now within the graphene layer, we find $(z=0)$

$$\phi(q_\perp, z=0) = -\frac{4\pi\sigma(q_\perp)}{(\varepsilon_1+\varepsilon_2)\,q_\perp} \Rightarrow$$

Comparison with calc. in (a) (on p. 1) shows:

$$\boxed{\frac{\varepsilon_1+\varepsilon_2}{2} = \varepsilon}$$