Theorie der Kondensierten Materie I WS 2012/2013

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1. BCS gap equation (10 + 30 = 40 Punkte)

The BCS gap equation reads

$$\Delta = \nu_0 V_0 \int_0^{\omega_D} d\xi \frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \tanh \frac{\sqrt{\xi^2+\Delta^2}}{2T} ,$$

where ν_0 is the density of states (assumed constant over the range of integration), $V_0 > 0$ describes the attractive interaction, $\xi = \epsilon - \mu$ is the energy of the electrons measured from the Fermi surface, T is the temperature and ω_D the Debye frequency. We assume that $\Delta \in \mathbb{R}$.

(a) Determine the critical temperature T_c for a transition to a superconducting state ($\Delta \neq 0$) from the condition that $\Delta(T_c) = 0$.

(b) Consider temperatures below but close to T_c, where the gap is small. The hierarchy of energy scales reads $\omega_D \gg T_c \sim \Delta(T=0) \gg \Delta(T \simeq T_c)$. Expand the right-hand side of the gap equation to first order in Δ^2, and thus find the temperature behavior of the gap $\Delta(T)$ for $T \lesssim T_c$.

2. Thermodynamics of BCS theory (15 + 15 + 30 = 60 Punkte)

(a) Determine the entropy of a superconductor at temperature T. The Bogoliuov quasi-particles constitute a Fermi gas with dispersion $E_k = \sqrt{\xi_k^2 + \Delta^2}$ where $\xi_k = \epsilon_k - \mu$. *Note*: a result from the first homework might be useful.

(b) Find an expression for the heat capacity $C_V(T)$ of a superconductor at temperature T in terms of E_k, T, Δ and the Fermi function $f(E_k)$.

(c) Determine the heat capacity $C_V(T)$ close to T_c. You have to use the temperature behavior of $\Delta(T)$ found in the previous exercise and expand the derivative of the Fermi function as $\partial_E f(E) \approx -\delta(E) - \frac{\pi^2}{6} T^2 \delta''(E)$. Find the size of the universal jump of the heat capacity across T_c:

$$\frac{\delta C_V}{C_V(T = T_c + 0^+)} ,$$

where $\delta C_V = C_V(T = T_c - 0^+) - C_V(T = T_c + 0^+)$ and 0+ stands for a small positive real number.