Karlsruhe Institute of Technology

Winter-Semester 2017/18

Moderne Theoretische Physik IIIa

# Statistische Physik

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Do 11:30-13:00, Lehmann Raum 022, Geb 30.22

http://www.tkm.kit.edu/lehre/

# Ideales Bose-Gas

Der Zustand ist charakterisiert lediglich durch $n_\lambda$

$$N = \sum_\lambda n_\lambda \qquad E = \sum_\lambda n_\lambda \epsilon_\lambda \qquad n_\lambda = 0, 1, 2, \ldots, \infty$$

**Bosonen**

$$Z_G(T, V, \mu) = \sum_n e^{-\beta(E_n - \mu N_n)} = \sum_{\{n_\lambda\}} e^{-\beta \sum_\lambda n_\lambda(\epsilon_\lambda - \mu)} \Rightarrow Z_G = \prod_\lambda Z_\lambda$$

$$Z_\lambda \equiv \sum_{n_\lambda = 0}^{\infty} e^{-\beta(\epsilon_\lambda - \mu) n_\lambda} = \frac{1}{1 - e^{-\beta(\epsilon_\lambda - \mu)}}$$

$\forall \lambda$ gilt $\epsilon_\lambda \geq \mu$

# Ideales Bose-Gas

## Allgemeine Relationen

$$Z_G(T,V,\mu) = \prod_\lambda \left[\frac{1}{1-e^{-\beta(\epsilon_\lambda-\mu)}}\right]$$

$$\Omega(T,V,\mu) = -k_\mathrm{B}T \ln Z_G(T,V,\mu) = k_\mathrm{B}T\sum_\lambda \ln\left[1-e^{-\beta(\epsilon_\lambda-\mu)}\right]$$

$$\langle N\rangle = -\frac{\partial\Omega}{\partial\mu}\Big|_{T,V} = \sum_\lambda \langle n_\lambda\rangle \qquad \langle n_\lambda\rangle = n_B(\epsilon_\lambda) = \frac{1}{e^{\beta(\epsilon_\lambda-\mu)}-1}$$

$$S = -\frac{\partial\Omega}{\partial T}\Big|_{V,\mu} = -k_\mathrm{B}\sum_\lambda \left[\langle n_\lambda\rangle\ln\langle n_\lambda\rangle - (1+\langle n_\lambda\rangle)\ln(1+\langle n_\lambda\rangle)\right]$$

$$U = \Omega + TS + \mu N = \sum_\lambda \epsilon_\lambda n_B(\epsilon_\lambda)$$

# Ideales Bose-Gas

## Freie nichtrelativistische Teilchen im Kasten

$$\Omega(T,V,\mu) = -k_\mathrm{B}T \ln Z_G(T,V,\mu) = k_\mathrm{B}T \sum_\lambda \ln\left[1 - e^{-\beta(\epsilon_\lambda - \mu)}\right]$$

$\lambda = \mathbf{p}, \sigma$ z.B. $^{7}$Li, $^{23}$Na,... Gesamtspin von Elektronen und Kern $\mathbf{F} = \mathbf{S} + \mathbf{I}$

z.B. $F = 1$

$$\epsilon_\lambda = \epsilon_\mathbf{p} = \frac{\mathbf{p^2}}{2m}$$

$$\nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$$ Zustandsdichte

# Ideales Bose-Gas

$$\Omega(T,V,\mu) = -k_\mathrm{B}T \ln Z_G(T,V,\mu) = k_\mathrm{B}T \sum_\lambda \ln\left[1 - e^{-\beta(\epsilon_\lambda-\mu)}\right]$$

$$\epsilon_\lambda = \epsilon_\mathbf{p} = \frac{\mathbf{p^2}}{2m} \qquad \nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$$ Zustandsdichte

$$z \equiv e^{\beta\mu}$$ Fugazität

$$\Omega(T,V,\mu) = k_\mathrm{B}T\,(2s+1)\,\ln[1-z] + k_\mathrm{B}T(2s+1)V\int_0^\infty d\epsilon\,\nu(\epsilon)\ln\left[1 - ze^{-\beta\epsilon}\right]$$

Der Beitrag von $\epsilon_\mathbf{p} = 0$ muss separat behandelt werden um den Limes $\mu \to 0$ richtig zu behandeln!

# Ideales Bose-Gas

$z \equiv e^{\beta\mu}$ Fugazität

$\nu(\epsilon)$ Zustandsdichte

$$\Omega(T,V,\mu) = k_B T\,(2s+1)\,\ln[1-z] + k_B T(2s+1)V \int_0^\infty d\epsilon\,\nu(\epsilon)\ln\left[1 - z e^{-\beta\epsilon}\right]$$

Für $\nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$ gilt

$$\Omega(T,V,\mu) = k_B T\,(2s+1)\,\ln[1-z] - k_B T(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(z)$$

$$g_{5/2}(z) \equiv -\frac{4}{\sqrt{\pi}}\int_0^\infty dx\,x^2\ln(1 - z e^{-x^2}) = \sum_{n=1}^\infty \frac{z^n}{n^{5/2}}$$

$$\lambda_T = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$$

Thermische Wellenlänge

# Ideales Bose-Gas

$z \equiv e^{\beta\mu}$ **Fugazität**

$$\Omega(T,V,\mu) = k_{\rm B}T\,(2s+1)\,\ln[1-z] + k_{\rm B}T(2s+1)V\int_0^\infty d\epsilon\,\nu(\epsilon)\ln\left[1-ze^{-\beta\epsilon}\right]$$

$$N = -\frac{\partial\Omega}{\partial\mu}\Big|_{T,V} = (2s+1)\,\frac{z}{1-z} + (2s+1)V\int_0^\infty d\epsilon\,\nu(\epsilon)n_B(\epsilon)$$

**Für** $\nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$ **gilt**

$$N = -\frac{\partial\Omega}{\partial\mu}\Big|_{T,V} = (2s+1)\,\frac{z}{1-z} + (2s+1)\frac{V}{\lambda_T^3}g_{3/2}(z)$$

$$g_{5/2}(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^{5/2}} \qquad g_{3/2}(z) \equiv z\frac{\partial}{\partial z}g_{5/2}(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^{3/2}} \qquad \lambda_{\rm T} = \sqrt{\frac{2\pi\hbar^2}{mk_{\rm B}T}}$$

# Ideales Bose-Gas

$$\Omega(T,V,\mu) = -k_{\rm B}T \ln Z_G(T,V,\mu) = k_{\rm B}T \sum_\lambda \ln\left[1 - e^{-\beta(\epsilon_\lambda - \mu)}\right]$$

$$U = \Omega + TS + \mu N = \sum_\lambda \epsilon_\lambda n_B(\epsilon_\lambda)$$ Kein Beitrag von $\epsilon_{\mathbf{p}} = 0$

Für $\nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$ gilt

$$U(T,V,\mu) = \frac{3}{2} k_{\rm B}T(2s+1)\frac{V}{\lambda_T^3}\, g_{5/2}(z)$$

$$g_{5/2}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^{5/2}}$$

$$\lambda_{\rm T} = \sqrt{\frac{2\pi\hbar^2}{mk_{\rm B}T}}$$

# Ideales Bose-Gas

## Hohe Temperaturen $\quad n\lambda_T^3 \ll 1$

$$\Omega(T,V,\mu) = k_{\mathrm{B}}T\,(2s+1)\,\ln[1-z] - k_{\mathrm{B}}T(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(z) \approx -k_{\mathrm{B}}T(2s+1)\frac{V}{\lambda_T^3}\,z$$

$$N = (2s+1)\,\frac{z}{1-z} + (2s+1)\frac{V}{\lambda_T^3}g_{3/2}(z) \approx (2s+1)\frac{V}{\lambda_T^3}\,z$$

$$U(T,V,\mu) = \frac{3}{2}\,k_{\mathrm{B}}T(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(z) \approx \frac{3}{2}\,k_{\mathrm{B}}T(2s+1)\frac{V}{\lambda_T^3}\,z$$

$$PV = -\Omega = k_{\mathrm{B}}TN \qquad\qquad U = \frac{3}{2}\,k_{\mathrm{B}}TN$$

Ideales klassisches Gas

$$g_{5/2}(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^{5/2}} \qquad g_{3/2}(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^{3/2}} \qquad \lambda_{\mathrm{T}} = \sqrt{\frac{2\pi\hbar^2}{mk_{\mathrm{B}}T}}$$

# Ideales Bose-Gas

## Hohe Temperaturen $n\lambda_T^3 \ll 1$

### Korrekturen zum klassischen idealen Gas

$$N \approx (2s+1)\frac{V}{\lambda_T^3}\left(z + \frac{z^2}{2^{3/2}}\right) \qquad z \approx \frac{n\lambda_T^3}{(2s+1)}\left(1 - \frac{1}{2^{3/2}}\frac{n\lambda_T^3}{(2s+1)}\right)$$

$$\Omega \approx -k_\mathrm{B}T(2s+1)\frac{V}{\lambda_T^3}\left(z + \frac{z^2}{2^{5/2}}\right) \qquad PV \approx k_\mathrm{B}TN\left(1 - \frac{1}{2^{5/2}}\frac{n\lambda_T^3}{(2s+1)}\right)$$

$$U \approx \frac{3}{2}k_\mathrm{B}T(2s+1)\frac{V}{\lambda_T^3}\left(z + \frac{z^2}{2^{5/2}}\right) \qquad U \approx \frac{3}{2}k_\mathrm{B}TN\left(1 - \frac{1}{2^{5/2}}\frac{n\lambda_T^3}{(2s+1)}\right)$$

$$g_{5/2}(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^{5/2}} \qquad g_{3/2}(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^{3/2}} \qquad \lambda_\mathrm{T} = \sqrt{\frac{2\pi\hbar^2}{mk_\mathrm{B}T}}$$

# Ideales Bose-Gas

$$\lambda_{\mathrm{T}} = \sqrt{\frac{2\pi\hbar^2}{mk_{\mathrm{B}}T}}$$

Hohe Temperaturen $n\lambda_T^3 \ll 1$

$$z \approx \frac{n\lambda_T^3}{(2s+1)} \ll 1 \quad \Rightarrow \quad \mu < -k_{\mathrm{B}}T$$

$$PV \approx k_{\mathrm{B}}TN\left(1 - \frac{1}{2^{5/2}}\frac{n\lambda_T^3}{(2s+1)}\right) < k_{\mathrm{B}}TN$$

Bose-Statistik ~ Anziehung

# Bose-Einstein-Kondensation

Tiefe Temperaturen $n\lambda_T^3 \gg 1$

$$N = -\frac{\partial\Omega}{\partial\mu}\Big|_{T,V} = (2s+1)\,\frac{z}{1-z} + (2s+1)V\int_0^\infty d\epsilon\,\nu(\epsilon)n_B(\epsilon)$$

Für $\nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$ gilt

$$N = (2s+1)\,\frac{z}{1-z} + (2s+1)\frac{V}{\lambda_T^3}g_{3/2}(z)$$

$$N_0 \equiv (2s+1)\,\frac{z}{1-z} = (2s+1)n_B(\epsilon=0)$$

Teilchenzahl im Grundzustand

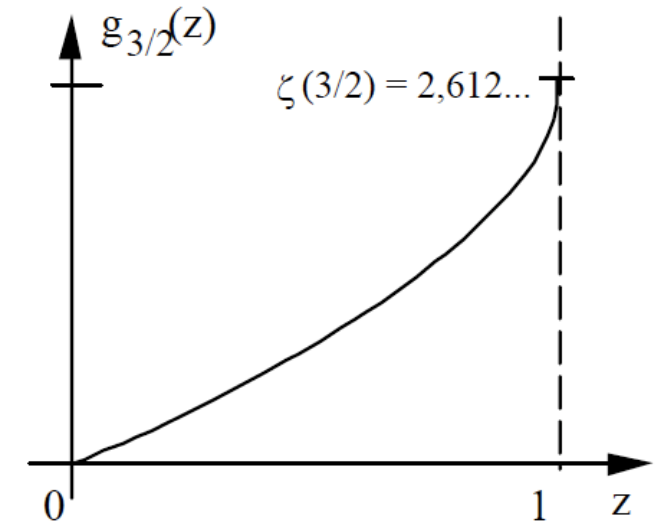
$$n \equiv \frac{N}{V} = \frac{N_0}{V} + (2s+1)\frac{1}{\lambda_T^3}g_{3/2}(z)$$

$$g_{3/2}(z) = \sum_{n=1}^\infty \frac{z^n}{n^{3/2}} \qquad \lambda_{\mathrm{T}} = \sqrt{\frac{2\pi\hbar^2}{mk_{\mathrm{B}}T}}$$

# Bose-Einstein-Kondensation

$$n \equiv \frac{N}{V} = \frac{N_0}{V} + (2s+1)\frac{1}{\lambda_T^3} g_{3/2}(z)$$

$$N_0 \equiv (2s+1)\,\frac{z}{1-z}$$




Für $\quad n < n_c(T) \equiv (2s+1)\frac{1}{\lambda_T^3} g_{3/2}(1)$ $\quad$ maximale Dichte nicht im Kondensat

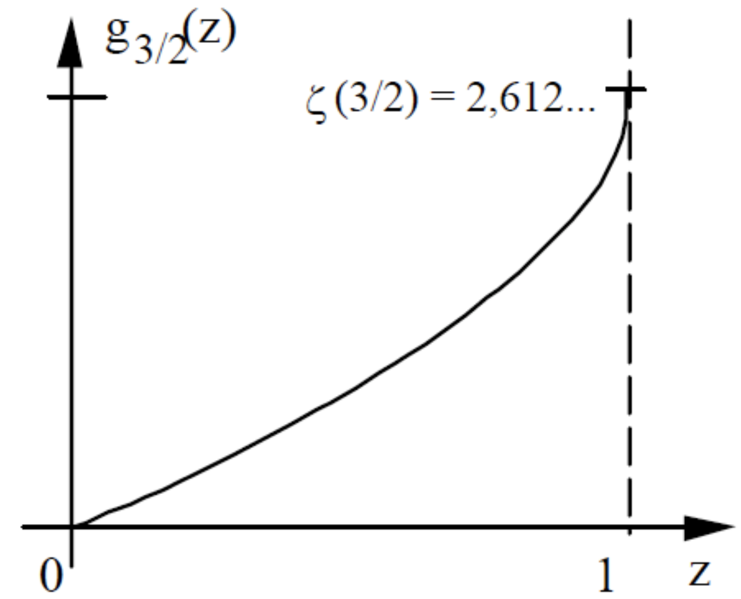
oder $\quad T > T_c \equiv \frac{2\pi\hbar^2}{mk_\mathrm{B}} \left(\frac{n}{(2s+1)g_{3/2}(1)}\right)^{2/3}$ $\quad \lambda_\mathrm{T} = \sqrt{\frac{2\pi\hbar^2}{mk_\mathrm{B}T}}$

normales Gas

$$\lim_{V\to\infty} \frac{N_0}{V} = 0 \qquad \lim_{V\to\infty} z < 1 \qquad \lim_{V\to\infty} \mu < 0$$

# Bose-Einstein-Kondensation

$$n \equiv \frac{N}{V} = \frac{N_0}{V} + (2s+1)\frac{1}{\lambda_T^3} g_{3/2}(z)$$




$$N_0 \equiv (2s+1)\,\frac{z}{1-z}$$

Für $\quad n > n_c(T) \equiv (2s+1)\frac{1}{\lambda_T^3} g_{3/2}(1)$

oder $\quad T < T_c \equiv \frac{2\pi\hbar^2}{m k_\mathrm{B}} \left( \frac{n}{(2s+1) g_{3/2}(1)} \right)^{2/3}$

$$\lambda_\mathrm{T} = \sqrt{\frac{2\pi\hbar^2}{m k_\mathrm{B} T}}$$

## Teil der Teilchen im Kondensat

$$\lim_{V\to\infty} \frac{N_0}{V} \equiv n_0 = n - n_c \qquad \lim_{V\to\infty} z = 1 \qquad \lim_{V\to\infty} \mu = 0$$

# Bose-Einstein-Kondensation

Für $T < T_c \equiv \frac{2\pi\hbar^2}{mk_\mathrm{B}} \left(\frac{n}{(2s+1)g_{3/2}(1)}\right)^{2/3}$

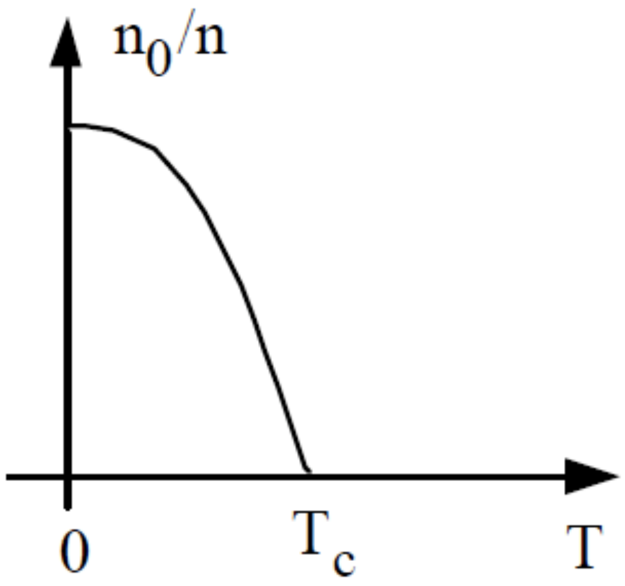
$$\lambda_\mathrm{T} = \sqrt{\frac{2\pi\hbar^2}{mk_\mathrm{B}T}}$$

$$n = n_0 + (2s+1)\frac{1}{\lambda_T^3}g_{3/2}(1) = n_0 + n_c(T)$$

$$n_c(T) \equiv (2s+1)\frac{1}{\lambda_T^3}g_{3/2}(1)$$

bei $T_c$ gilt $\quad n = n_c(T_c)$

$$\frac{n_0}{n} = 1 - \frac{n_c(T)}{n} = 1 - \frac{n_c(T)}{n_c(T_c)} = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$



# Bose-Einstein-Kondensation

## Tiefe Temperaturen

$$N = -\frac{\partial\Omega}{\partial\mu}\Big|_{T,V} = (2s+1)\,\frac{z}{1-z} + (2s+1)V\int_0^\infty d\epsilon\,\nu(\epsilon)n_B(\epsilon)$$

Für beliebige Zustandsdichte $\nu(\epsilon)$

die Frage ist ob $\int_0^\infty d\epsilon\,\nu(\epsilon)n_B(\epsilon,\mu=0) = \int_0^\infty d\epsilon\,\frac{\nu(\epsilon)}{e^{\beta\epsilon}-1}$

konvergiert

$$n_c(T) \equiv (2s+1)\int_0^\infty d\epsilon\,\frac{\nu(\epsilon)}{e^{\beta\epsilon}-1} < \infty$$

BEK $n_c(T_c) = n$

divergiert

$$n_c(T) \equiv (2s+1)\int_0^\infty d\epsilon\,\frac{\nu(\epsilon)}{e^{\beta\epsilon}-1} \to \infty$$

keine BEK

# Bose-Einstein-Kondensation

## Erfolgt nicht in 2D oder 1D

(für $\epsilon_\lambda \sim p^2$)

in 2D gilt $\nu(\epsilon) \propto \epsilon^0$

in 1D gilt $\nu(\epsilon) \propto \epsilon^{-1/2}$

$$n_c(T) \equiv (2s+1) \int_0^\infty d\epsilon \, \frac{\nu(\epsilon)}{e^{\beta\epsilon} - 1} \to \infty$$

Makroskopische Besetzung des Grundzustandes nicht nötig

# Bose-Einstein-Kondensation

## Aus Wikipedia

This state of matter was first predicted by Satyendra Nath Bose and Albert Einstein in 1924–25. Bose first sent a paper to Einstein on the quantum statistics of light quanta (now called photons). Einstein was impressed, translated the paper himself from English to German and submitted it for Bose to the Zeitschrift für Physik which published it. Einstein then extended Bose's ideas to material particles (or matter) in two other papers.

The Einstein manuscript, believed to be lost, was found in a library at Leiden University in 2005.

# The Nobel Prize in Physics 2001

"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

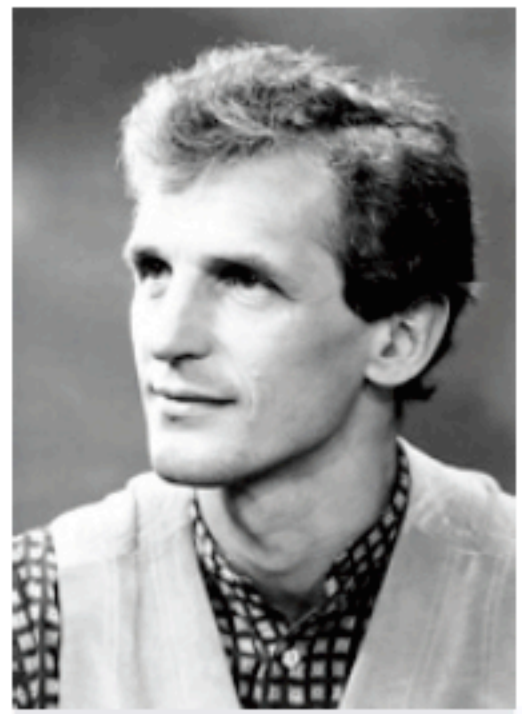
**Eric A. Cornell**

1/3 of the prize

USA

University of Colorado, JILA
Boulder, CO, USA

b. 1961



**Wolfgang Ketterle**

1/3 of the prize

Federal Republic of Germany

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

b. 1957

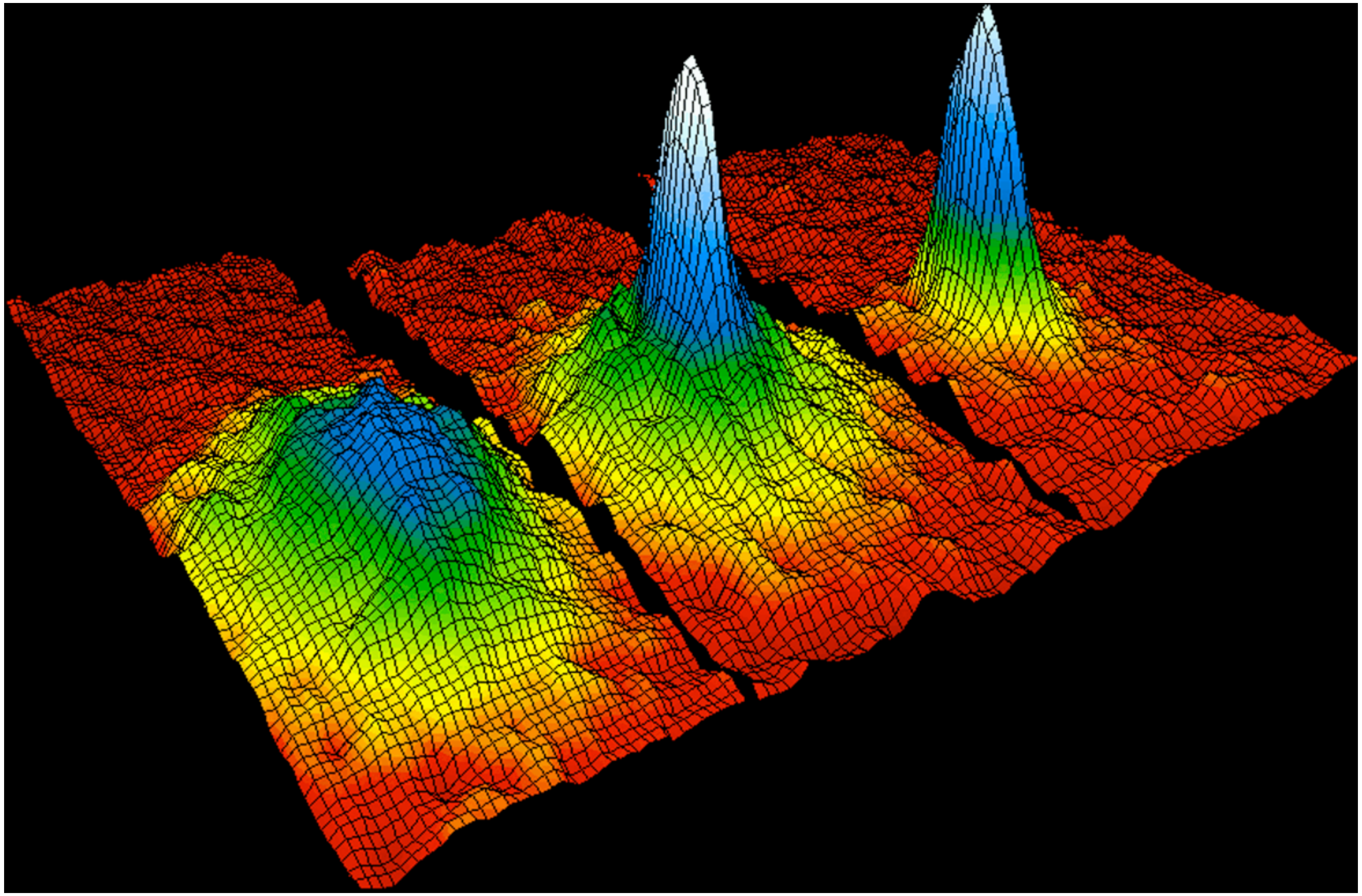
**Carl E. Wieman**

1/3 of the prize

USA

University of Colorado, JILA
Boulder, CO, USA

b. 1951



Geschwindigkeitsverteilung im BEC

# Bose-Einstein-Kondensation

## Chemisches Potential $T < T_c$

$$n = \frac{(2s+1)z}{V(1-z)} + (2s+1)\int_0^\infty d\epsilon\,\nu(\epsilon)n_B(\epsilon) = \frac{(2s+1)z}{V(1-z)} + n_{\epsilon>0}(T)$$

für $T < T_c$ gilt

$$n = \frac{(2s+1)z}{V(1-z)} + n_c(T) \quad \Rightarrow \quad \frac{(2s+1)z}{V(1-z)} = n - n_c(T) = n_0(T) > 0$$

$$\frac{1-z}{z} = z^{-1} - 1 = e^{-\beta\mu} - 1 = \frac{2s+1}{V n_0(T)}$$

$$-\beta\mu \propto \frac{1}{V n_0(T)} \xrightarrow{V\to\infty} 0$$

$$\Delta p \sim 1/L$$

$$\Delta\epsilon_{\epsilon=0} \sim 1/L^2 \sim 1/V^{2/3}$$

$$\Delta\epsilon_{\epsilon=0} \sim 1/L^2 \sim V^{-2/3} \gg |\mu|$$

nur $\epsilon = 0$ makroskopisch besetzt

# Bose-Einstein-Kondensation

## Chemisches Potential $T > T_c$

$$n = (2s+1) \int_0^\infty d\epsilon\, \nu(\epsilon) n_B(\epsilon, z)$$

$$n = (2s+1) \frac{1}{\lambda_T^3} g_{3/2}(z)$$

$$\nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$$

$$\frac{\partial n}{\partial T} = 0 \quad \Rightarrow \quad \frac{\partial z}{\partial T} = \frac{3z}{2T} \frac{g_{3/2}(z)}{g_{1/2}(z)}$$

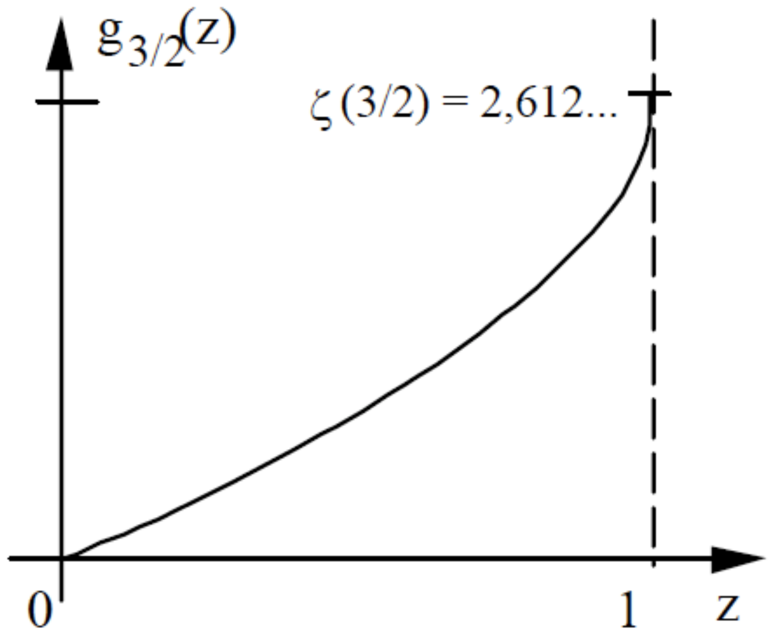
$$g_{3/2}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^{3/2}}$$

# Bose-Einstein-Kondensation

## Chemisches Potential $T > T_c$

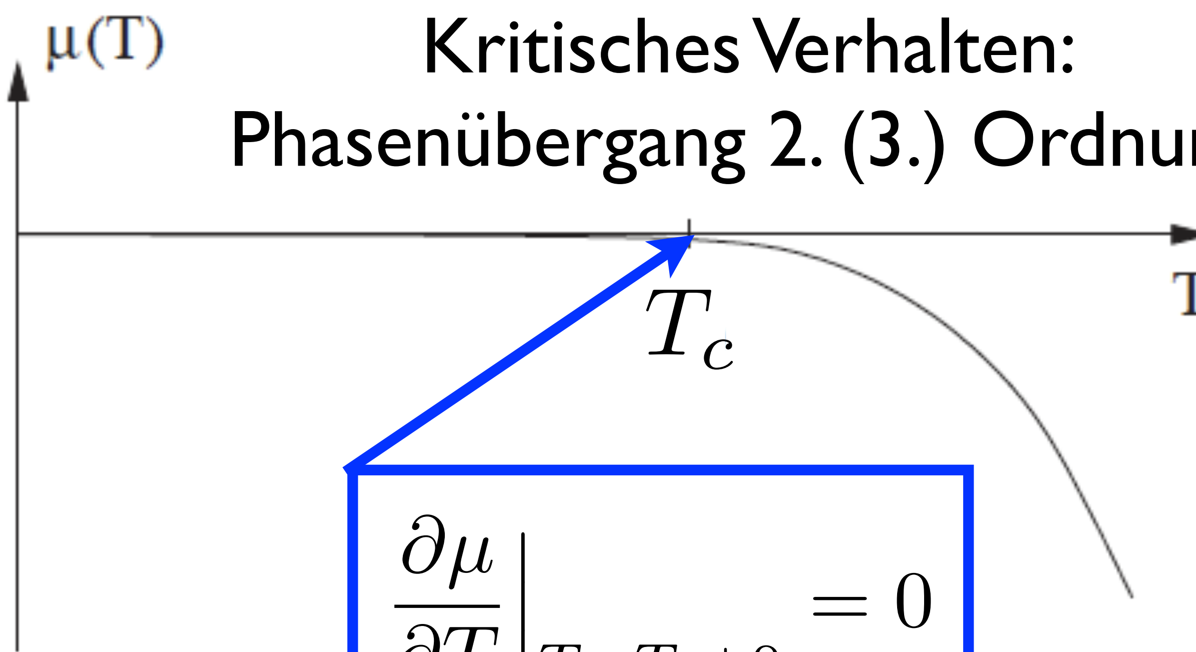
$$n = (2s+1)\int_0^\infty d\epsilon\, \nu(\epsilon) n_B(\epsilon, z)$$

$$n = (2s+1)\frac{1}{\lambda_T^3} g_{3/2}(z) \quad \text{für} \quad \nu(\epsilon) = \frac{m^{3/2}\epsilon^{1/2}}{\sqrt{2}\pi^2\hbar^3}$$




$$g_{3/2}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^{3/2}}$$

Kritisches Verhalten:
Phasenübergang 2. (3.) Ordnung




$$\left.\frac{\partial \mu}{\partial T}\right|_{T=T_c+0} = 0$$

$$\left.\frac{\partial^2 \mu}{\partial T^2}\right|_{T=T_c+0} < 0$$

# Ideales Bose-Gas

## Druck

$$\Omega(T,V,\mu) = k_\mathrm{B}T\,(2s+1)\,\ln[1-z] - k_\mathrm{B}T(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(z)$$

$$P = -\frac{\Omega}{V} = -k_\mathrm{B}T\,(2s+1)\frac{\ln[1-z]}{V} + k_\mathrm{B}T(2s+1)\frac{1}{\lambda_T^3}\,g_{5/2}(z)$$

$$\lim_{V\to\infty}\frac{\ln[1-z]}{V} = 0 \quad \Rightarrow \quad P = k_\mathrm{B}T(2s+1)\frac{1}{\lambda_T^3}\,g_{5/2}(z)$$

$$g_{5/2}(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^{5/2}}$$

$$\lambda_\mathrm{T} = \sqrt{\frac{2\pi\hbar^2}{mk_\mathrm{B}T}}$$

# Ideales Bose-Gas

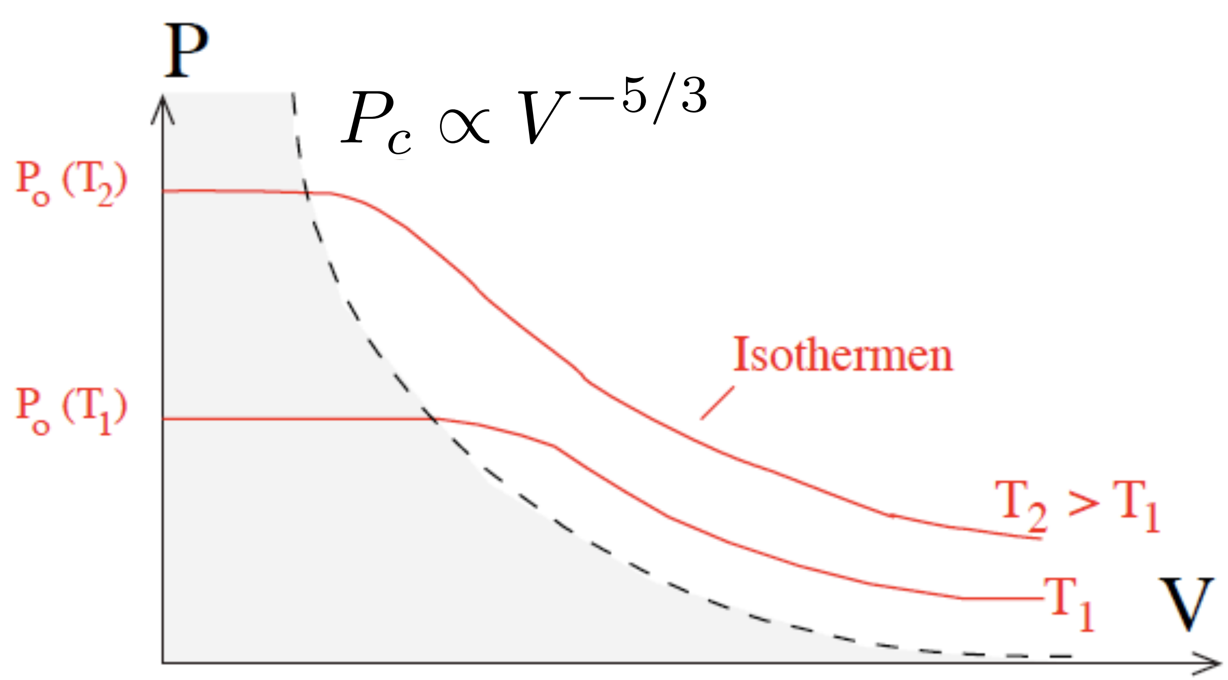
## Druck

$$P = k_{\mathrm{B}} T (2s+1) \frac{1}{\lambda_T^3}\, g_{5/2}(z)$$

$$T_c \equiv \frac{2\pi\hbar^2}{m k_{\mathrm{B}}} \left( \frac{n}{(2s+1) g_{3/2}(1)} \right)^{2/3}$$

$$P_c = k_{\mathrm{B}} T_c (2s+1) \frac{1}{\lambda_{T_c}^3}\, g_{5/2}(1)$$

$$P_c \propto n^{5/3} \propto V^{-5/3}$$




$$g_{5/2}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^{5/2}}$$

$$\lambda_{\mathrm{T}} = \sqrt{\frac{2\pi\hbar^2}{m k_{\mathrm{B}} T}}$$

# Bose-Einstein-Kondensation

## Entropie

$$\Omega(T,V,\mu) = k_\mathrm{B}T\,(2s+1)\,\ln[1-z] - k_\mathrm{B}T(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(z)$$

$$N = (2s+1)\,\frac{z}{1-z} + (2s+1)\frac{V}{\lambda_T^3}g_{3/2}(z) \qquad \lim_{V\to\infty}\frac{\ln[1-z]}{V} = 0$$

für $T > T_c$ $\qquad S = -\frac{\partial\Omega}{\partial T}\Big|_{V,\mu} = \frac{5}{2}\,k_\mathrm{B}(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(z) - k_\mathrm{B}N\ln z$

für $T < T_c$ $\qquad S = -\frac{\partial\Omega}{\partial T}\Big|_{V,\mu} = \frac{5}{2}\,k_\mathrm{B}(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(1) \propto T^{3/2}$

3. Hauptsatz

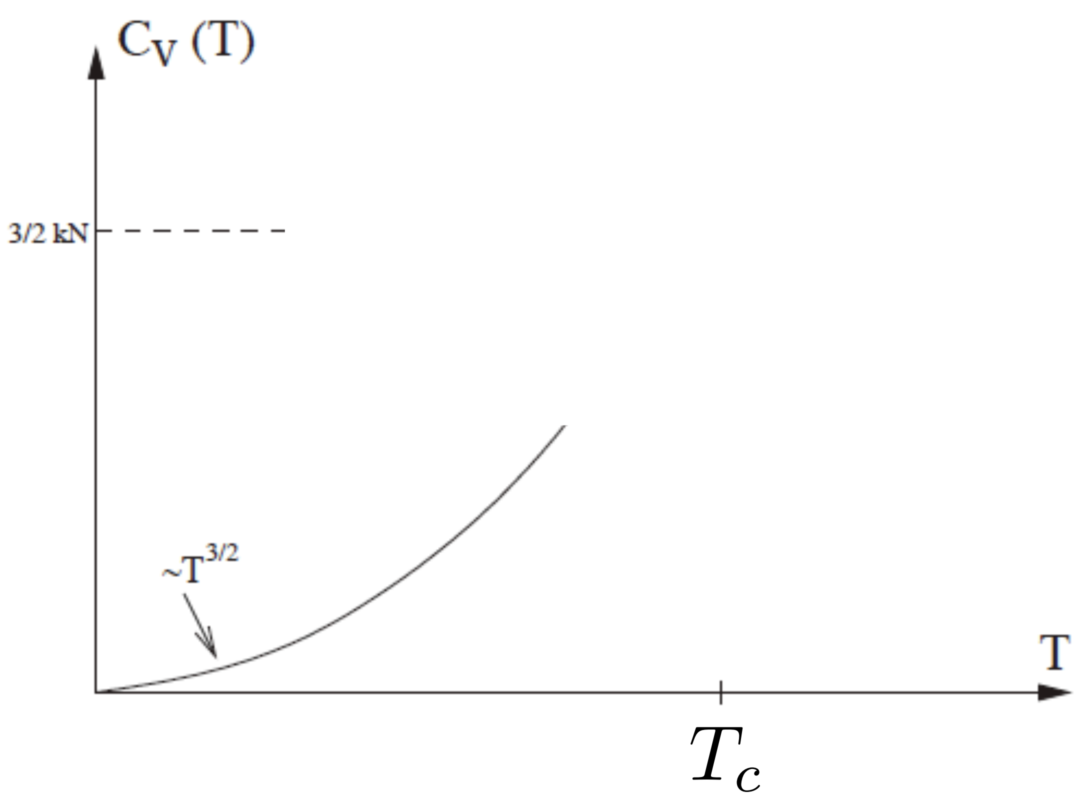
# Bose-Einstein-Kondensation

## Wärmekapazität

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} = T\left(\frac{\partial S}{\partial T}\right)_{V,N} = T\left(\frac{\partial S(T,V,\mu(N,T))}{\partial T}\right)_{V,N}$$

für $T < T_c$ gilt $\mu = 0 = const.$ und $S = \frac{5}{2}\,k_\mathrm{B}(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(1) \propto T^{3/2}$

für $T < T_c$ gilt $C_V = \frac{15}{4}\,k_\mathrm{B}(2s+1)\frac{V}{\lambda_T^3}\,g_{5/2}(1) \propto T^{3/2}$



# Bose-Einstein-Kondensation

## Wärmekapazität

$$U(T,V,\mu) = \frac{3}{2}\, k_{\mathrm{B}} T (2s+1) \frac{V}{\lambda_T^3}\, g_{5/2}(z)$$

$$\begin{aligned} C_V &= \left( \frac{\partial U(T,V,\mu(N,T))}{\partial T} \right)_{V,N} \\ &= \frac{15}{4}\, k_{\mathrm{B}} (2s+1) \frac{V}{\lambda_T^3}\, g_{5/2}(z) - \frac{3}{2} \left( \frac{\mu}{T} \right) \frac{V}{\lambda_T^3}\, g_{3/2}(z) + \frac{3}{2} \left( \frac{\partial \mu}{\partial T} \right) \frac{V}{\lambda_T^3}\, g_{3/2}(z) \end{aligned}$$

$$\left.\frac{\partial \mu}{\partial T}\right|_{T=T_c} = 0$$

$$\left.\frac{\partial^2 \mu}{\partial T^2}\right|_{T=T_c} < 0$$