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THEORETICAL OPTICS: EXERCISE SHEET 10

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1. Measuring temporal coherence with a Michelson interferometer

Consider the Michelson interferometer presented Fig. 1. With black we depict perfectly reflecting mirrors. With brown-pink we depict a $50\% - 50\%$ beam-splitter. Before reaching the detector, the two beams (dashed blue and magenta) merge into a single beam (dashed red).

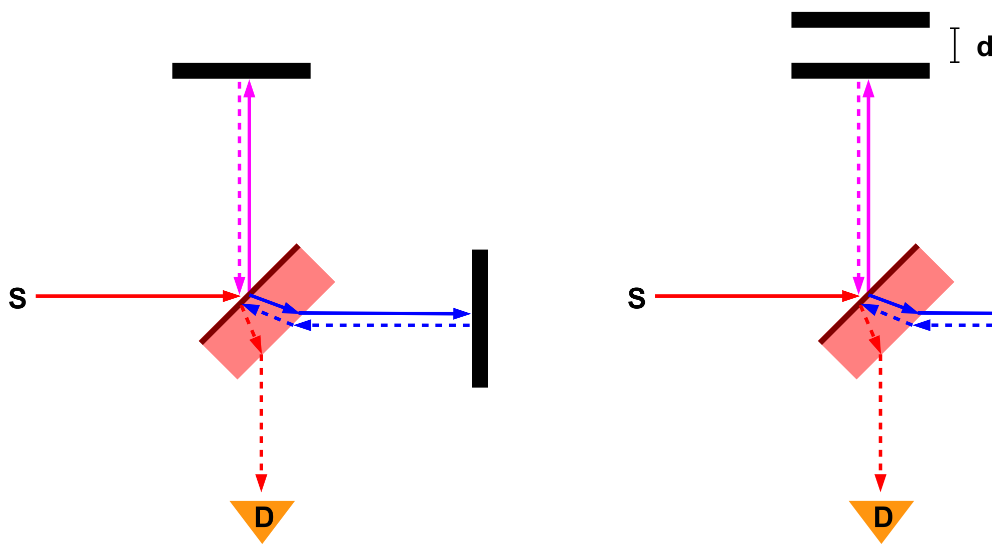

FIG. 1: Michelson interferometer. The initial red beam is split into a reflected magenta beam (solid magenta) and a transmitted blue beam (solid blue) by a $50\% - 50\%$ beam-splitter. The two beams are subsequently reflected by ideal mirrors (dashed magenta and blue beams). The reflected beams merge into a single red beam (dashed) after arriving to the beam-splitter. The intensity of the final beam (dashed red) is recorded by a detector. (Left) The position of the mirrors has been tuned so that the phase difference originating from the different optical paths of the two beams to be zero. (Right) The position of the upper mirror is shifted so to introduce a time delay $\tau = 2d/c$ to the magenta beam.

a. Calculate the electric field intensity at the detector, for the setup of Fig. 1 (left), in the case of a perfectly coherent beam of frequency ω_0. Assume we have tuned the positions of the mirrors, so that the two beams see no phase difference due to the different optical paths. (5 points)

b. Consider the setup of Fig. 1 (right) and assume a partially coherent initial beam with

$$\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau) = \langle \mathcal{E}(\boldsymbol{r}_1, t+\tau)\mathcal{E}^*(\boldsymbol{r}_2, t)\rangle \ . \tag{1}$$

Express the average intensity at the detector $\bar{I}(\tau)$ and the related fringe visibility

$$\mathcal{V} = \frac{\bar{I}_{\max} - \bar{I}_{\min}}{\bar{I}_{\max} + \bar{I}_{\min}} \,, \tag{2}$$

using the complex degree of self-coherence $\gamma(\boldsymbol{r}, \boldsymbol{r}, \tau) = \Gamma(\boldsymbol{r}, \boldsymbol{r}, \tau)/\Gamma(\boldsymbol{r}, \boldsymbol{r}, 0)$. (6 points)

c. Using Wiener-Khinchin theorem, calculate the power spectrum $S(\omega)$ of the partially coherent source $\boldsymbol{S}$, if the detector records the following profile

$$\Gamma(\boldsymbol{r},\boldsymbol{r},\tau) = \mathcal{A}e^{-i\omega_0\tau-(\tau/\tau_0)^2/2}\,, \tag{3}$$

where $\mathcal{A}$ a constant. (5 points)

d. Calculate the coherence time

$$\tau_c = \sqrt{\frac{1}{N}\int_{-\infty}^{+\infty}\tau^2|\gamma(\boldsymbol{r},\boldsymbol{r},\tau)|^2\,d\tau} \qquad \text{with} \qquad N = \int_{-\infty}^{+\infty}|\gamma(\boldsymbol{r},\boldsymbol{r},\tau)|^2\,d\tau\,, \tag{4}$$

for the profile of **1c.**. (4 points)

2. Young's double slit experiment with partially coherent sources

Consider two partially coherent light sources located at $\boldsymbol{O}$ and $\boldsymbol{O'}$, Fig. 2.

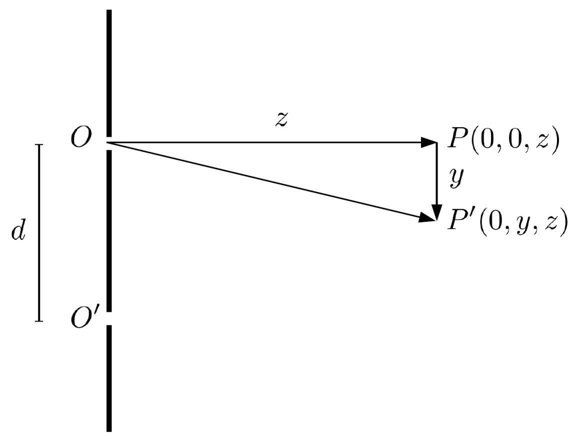

FIG. 2: Young' double slit experiment with partially coherent light sources located at $\boldsymbol{O}$ and $\boldsymbol{O'}$. For the particular setup consider $\boldsymbol{z >> d, y}$.

a. Express the average intensity at the observation point $\boldsymbol{P'}$, as a function of the complex degree of mutual-coherence (5 points)

$$\gamma(\boldsymbol{r}_1,\boldsymbol{r}_2,\tau) = \frac{\Gamma(\boldsymbol{r}_1,\boldsymbol{r}_2,\tau)}{\sqrt{\Gamma(\boldsymbol{r}_1,\boldsymbol{r}_1,0)\Gamma(\boldsymbol{r}_2,\boldsymbol{r}_2,0)}}\,. \tag{5}$$

b. Calculate the power spectrum $S(\omega)$ if

$$\Gamma(\boldsymbol{r}_1,\boldsymbol{r}_2,\tau) = \mathcal{A}e^{-i\omega_0\tau-|\tau|/\tau_0}\,, \tag{6}$$

where $\mathcal{A} = \sqrt{\Gamma(\boldsymbol{r}_1,\boldsymbol{r}_1,0)\Gamma(\boldsymbol{r}_2,\boldsymbol{r}_2,0)}$. (5 points)