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THEORETICAL OPTICS: EXERCISE SHEET 6

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1. Spherical waves - Properties of Fresnel zones

Consider a light source located at $\boldsymbol{S}$, emitting spherical electromagnetic waves (Fig. 1) at wave-length λ. One can retrieve the electromagnetic wave at the observation point $\boldsymbol{P}$ using Huygens principle and the construction of the so-called Fresnel zones Z_n.

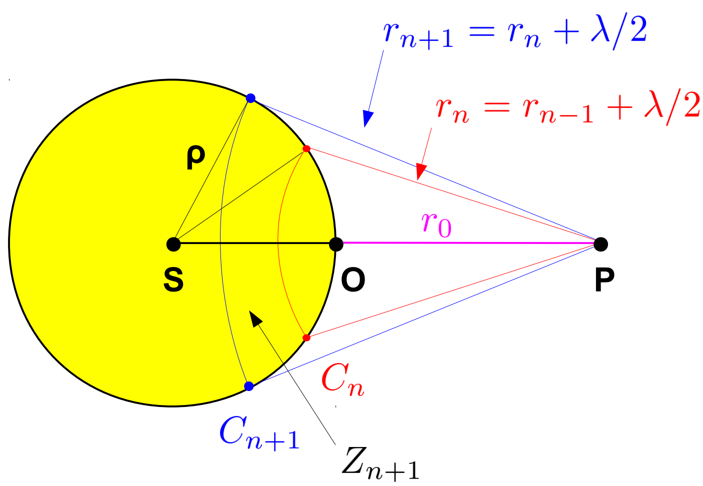

FIG. 1: A light source of spherical waves is located at $\boldsymbol{S}$. For retrieving the electromagnetic field at the observation point $\boldsymbol{P}$ one can construct Fresnel's zones which, according to Huygens' principle, are related to the secondary wavelets.

a. Verify that the electric field $\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}_0 e^{i(kr-\omega t)}/r$, with $\omega = ck$, is a solution of the wave equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \boldsymbol{\mathcal{E}}}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial \boldsymbol{\mathcal{E}}}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \boldsymbol{\mathcal{E}}}{\partial\varphi^2} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{\mathcal{E}}}{\partial t^2} = 0\,, \tag{1}$$

where (r,θ,φ) denote the spherical coordinates. Based on Maxwell's equation $\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{E}} = 0$ determine the polarization of $\boldsymbol{\mathcal{E}}_0$. (3 points)

b. Calculate the area A_n of the n-th Fresnel zone. Show that in the limit $\lambda/r_0 << 1$ and for small n, Fresnel's zones have approximately the same area (5 points)

$$A = \frac{\pi\lambda\rho r_0}{\rho + r_0}\,. \tag{2}$$

c. Assume that $\mathcal{E}_n \simeq -(\mathcal{E}_{n+1} + \mathcal{E}_{n-1})/2$. Show that for the particular approximation the total electric field at the observation point $\boldsymbol{P}$, becomes (4 points)

$$\mathcal{E}(\boldsymbol{P}) \simeq \frac{\mathcal{E}_1(\boldsymbol{P})}{2}\,. \tag{3}$$

2. Fresnel diffraction by a disc

Consider a light source located at $\boldsymbol{S}$, emitting spherical electromagnetic waves (Fig. 2) at wave-length λ. A non-transparent disc of radius R is placed between the source and the observation point $\boldsymbol{P}$.

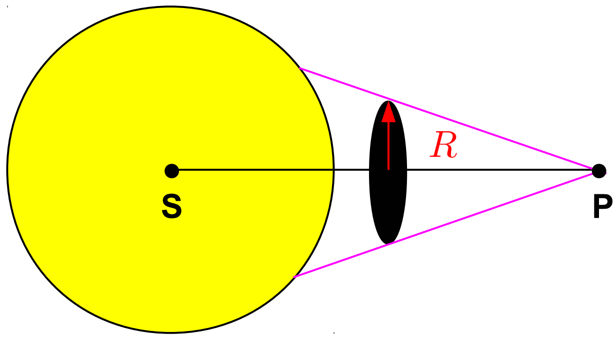

FIG. 2: A light source of spherical waves is located at $\boldsymbol{S}$. A non-transparent disc of radius R is placed between the source point $\boldsymbol{S}$ and the observation point $\boldsymbol{P}$.

a. Determine the minimum disc radius for which diffraction effects are observed. (3 points)

b. By considering that the Fresnel zones have approximately equal areas, calculate the number of Fresnel zones which are blocked when the disc is present. (3 points)

3. Fresnel diffraction by a ring

Consider a source of spherical electromagnetic waves, located at $\boldsymbol{S}$, originating from a He-Ne laser beam emitted at wave-length $\lambda = 632.8nm$. Between the source point $\boldsymbol{S}$ and the observation point $\boldsymbol{P}$ we equidistantly ($d = 1m$) place a circular ring (Fig. 3) of radii $R_1 \neq R_2$.

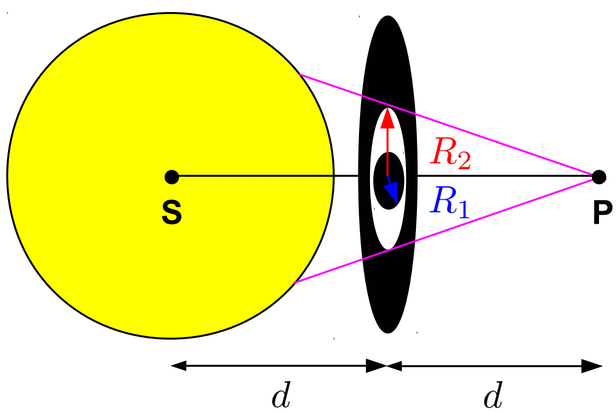

FIG. 3: A light source of spherical waves is located at $\boldsymbol{S}$. A ring of radii $R_1 \neq R_2$ is located between the source point $\boldsymbol{S}$ and the observation point $\boldsymbol{P}$. The outer non-transparent (black) part of the ring extends to infinity.

a. Calculate the minimum values for the radii R_1 and R_2 in order to have a bright spot at $\boldsymbol{P}$. What is the value of the electric field at the observation point $\boldsymbol{P}$ in this case? (4 points)

b. Calculate the minimum values for the radii R_1 and R_2 in order to have a dark spot at $\boldsymbol{P}$. (3 points)

4. Fresnel zone plate

The natural generalization of a ring is termed Fresnel zone plate (Fig. 4) and consists of alternating transparent (T) and non-transparent (N) regions, with radii R_n. The main use of a Fresnel zone plate is to filter out the odd or even Fresnel zones depending on the sequence of the regions NTNTNTN... or TNTNTNT..., respectively.

Show that

$$\frac{1}{\rho_0} + \frac{1}{r_0} \simeq \frac{n\lambda}{R_n^2}\,, \tag{4}$$

where ρ_0 and r_0 correspond to the distances of the Fresnel plate from $\boldsymbol{S}$ and $\boldsymbol{P}$, respectively (Fig. 5). The wavelength is denoted by λ. To retrieve this equation you have to consider the case $R_n << \rho_0, r_0$. (5 points)

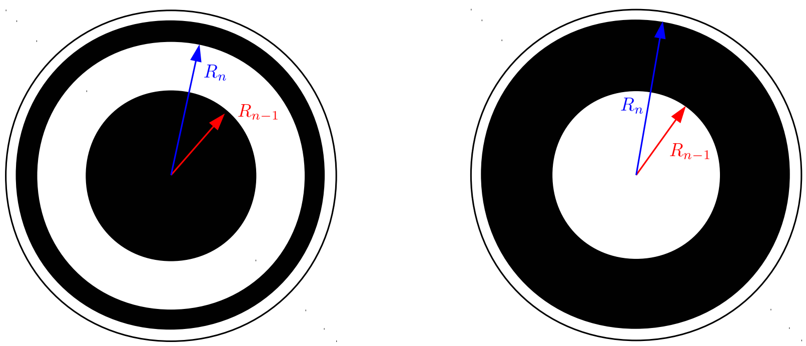

FIG. 4: Left: A Fresnel zone plate filtering out the odd Fresnel zones. Right: A Fresnel zone plate filtering out the even Fresnel zones. White corresponds to the transparent (T) regions. Black corresponds to the non-transparent (N) regions.

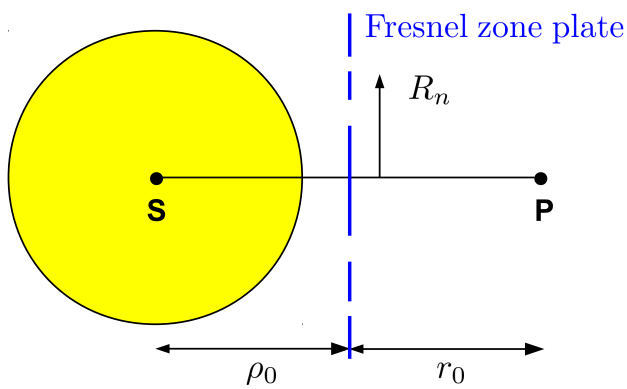

FIG. 5: Geometry of the diffraction situation under consideration. A Fresnel zone plate is placed between a source point $\boldsymbol{S}$ and an observation point $\boldsymbol{P}$.