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THEORETICAL OPTICS: EXERCISE SHEET 7

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1. Fresnel and Fraunhofer diffraction of spherical and plane waves

Consider a point source located at $\boldsymbol{S}$, emitting spherical electromagnetic waves of wave-length $\lambda = 2\pi/k$.

a. Calculate the electric field at the observation point $\boldsymbol{P}$ depicted in Fig. 1, using the scalar diffraction formula

$$\mathcal{E}(\boldsymbol{P}) = \frac{k}{2\pi i}\int_A \mathcal{E}(\boldsymbol{A})\frac{e^{ikr}}{r}\,dA\,, \tag{1}$$

where $\boldsymbol{A}$ denote the points of the wavefront and $r \equiv (AP)$. Only a part of the total wavefront contributes to the electric field due to the presence of the disc and the aperture of finite radii R_1 and R_2, respectively. The distance l, separating the centers of the disc and the aperture, is considered to be significantly smaller than the distance of the source $\boldsymbol{S}$ from the aperture. (3 points)

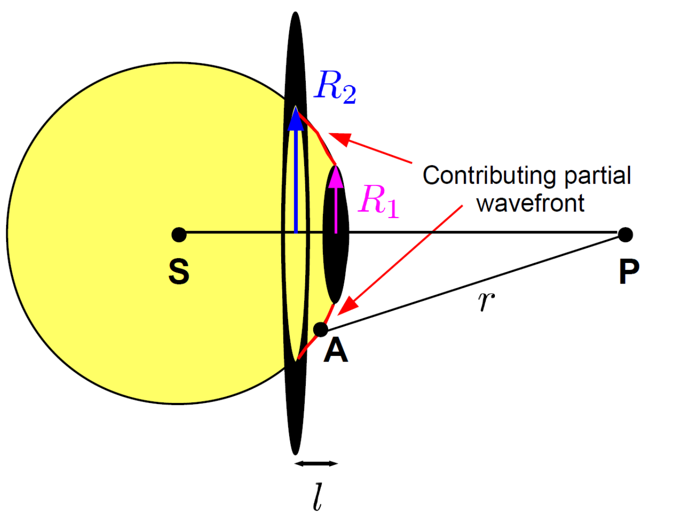

FIG. 1: Diffraction of spherical waves on a disc of radius R_1 combined with a circular aperture of radius R_2. Only a part of the spherical wavefront contributes to the electric field intensity at the observation point $\boldsymbol{P}$.

b. Consider a planar wavefront incident to a ring aperture depicted in Fig. 2. By employing Eq. (1), calculate the electric field $\mathcal{E}(\boldsymbol{P})$. Show that $\mathcal{E}(\boldsymbol{P})$ for a planar wavefront can be alternatively obtained from the result of **1a.**, by placing the point source $\boldsymbol{S}$ at an infinite distance from the aperture. (3 points)

c. Apply the Fresnel approximation $r \simeq r_0 + R^2/2r_0$ to the result of **1b.** (Fig. 2). (2 points)

d. Apply the "far-field" Fraunhofer approximation to the result of **1b.**. According to this approximation the radial distance R is completely negligible compared to r_0 (Fig. 2). Which is the minimum distance r_0 in order to observe a Fraunhofer pattern? (3 points)

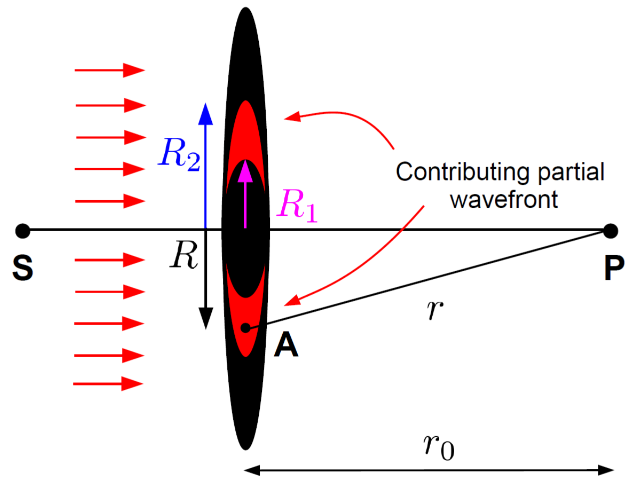

FIG. 2: Diffraction of plane waves on a screen consisting of a disc of radius R_1 and a cocentric circular aperture of radius R_2. Only a part of the planar wavefront contributes to the electric field intensity at the observation point $\boldsymbol{P}$.

2. Fresnel and Fraunhofer diffraction on a rectangular aperture

Consider a planar electromagnetic wavefront of wave-length $\lambda = 2\pi/k$ generated by a point source $\boldsymbol{S}$ placed in an infinite distance from the aperture (Fig. 3).

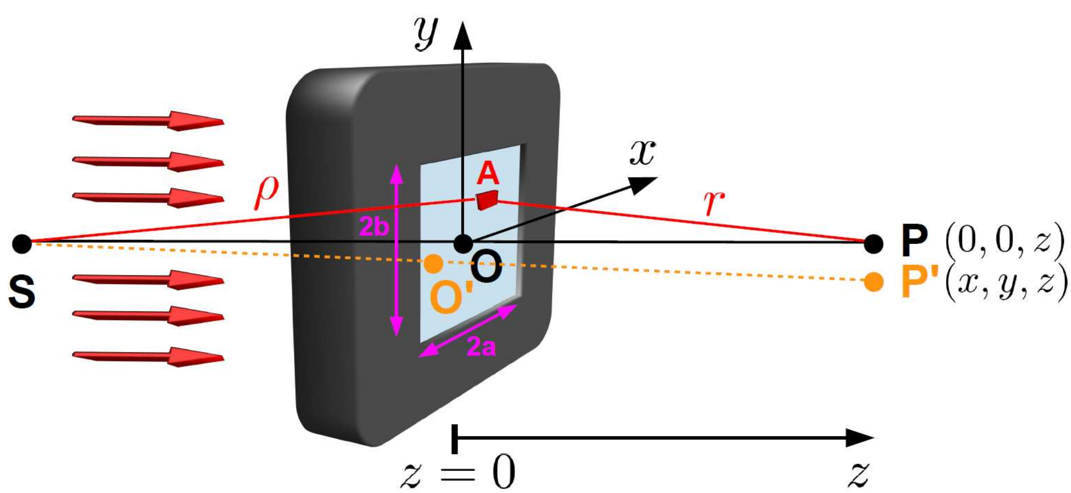

FIG. 3: Diffraction of plane waves on an rectangular aperture.

a. Using Eq. (1), calculate the electric field at the observation point $\boldsymbol{P}'(x, y, z)$ of Fig. 3, within the Fresnel approximation. For this calculation consider a new coordinate system with its origin at the point $\mathbf{O}'$, changing accordingly the integration limits. (4 points)

b. Consider $a = 1mm$, $b = 2mm$, $\lambda = 500nm$ and $z = 4m$. Calculate the ratio of intensities $I(\boldsymbol{P}')/I(\boldsymbol{P})$ for a point $\boldsymbol{P}'(-0.1mm, -0.1mm, 4m)$. (3 points)

c. Using Eq. (1), calculate the electric field at the observation point $\boldsymbol{P}'(x, y, z)$ of Fig. 3, within the Fraunhofer approximation. (3 points)

3. Diffraction grating

Consider a planar electromagnetic wavefront of wave-length $\lambda = 2\pi/k$ generated by a point source $\boldsymbol{S}$ placed in infinite distance from a screen that consists of N identical apertures diplaced by d (Fig. 4).

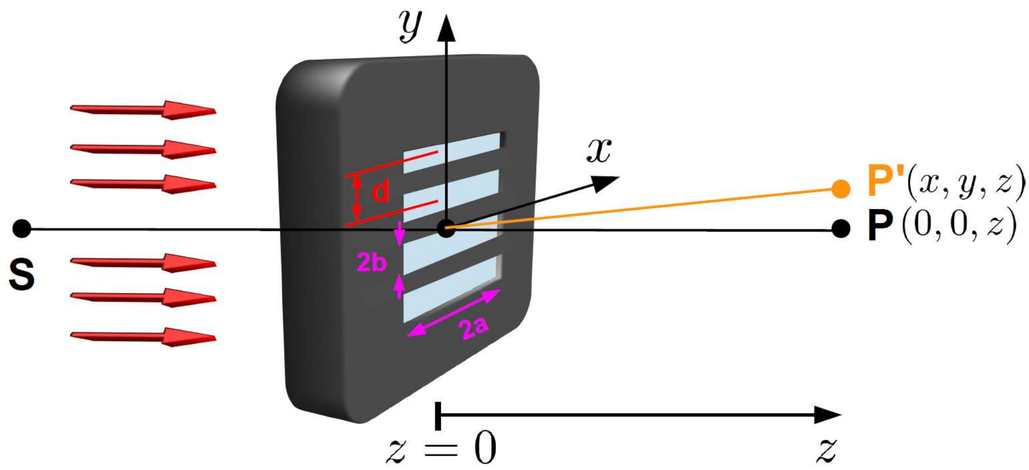

FIG. 4: Diffraction of plane waves on a screen consisting of an array of identical apertures.

a. Calculate the electric field at the observation point $\boldsymbol{P}'(x, y, z)$ within the Fraunhofer approximation. Show that the electric field intensity is a product of two terms, the first originating from the diffraction profile of a single aperture and the second from the interference of the N apertures. (3 points)

b. Determine the condition for observing intensity peaks. (3 points)

c. Consider that $a \to \infty$ (indepence on x-coordinate), $b = 1mm$, $\lambda = 500nm$, $d = 10mm$ and $N = 50$. Determine the number of maxima within the first principal peak. (3 points)